

Name:

MATH 506 Number Theory – **Exam II**
Wednesday March 31, 2004

Check that that you have all three pages - note that page two is on the back of page one

1. (12 points) Let $a = 2^3 3^5 5^7$ and $b = 2^7 3^2 17^3$. Find the following:

(a) The prime factorization of (a, b)

(b) The prime factorization of $[a, b]$

(c) The value of e if $3^e \parallel a^3 b^5$

(d) The value of f if $3^f \parallel (a^3 + b^5)$

2. (16 points) (a) Evaluate $\tau(280)$.

(b) Evaluate $\sigma(280)$.

(c) What prime factorizations are possible for n if $\tau(n) = 8$?

(d) What is the smallest n with $\tau(n) = 8$?

3. (8 points) Give a proof that there are infinitely many primes.

4. (10 points) Find all the right-angled triangles with coprime integer sides and base of length:

(a) 28

(b) 51

5. (8 points) Use induction to prove that $5^{2^n} \equiv 1 \pmod{2^{n+2}}$ for all positive integers n .

6. (6 points) Sieve out the primes from 271 to 288. Primes =

271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288

Multiples of which numbers had to be sifted out?

7. (8 points) (a) Find the missing digit ? if 1260054?414782 has least residue 1 (mod 11).

(b) Find h if $5^h \parallel 13051798632519117325175$

8. (24 points) Circle True (T) or False (F).

T F (a) If $p^5 \parallel ab$ and $p^2 \parallel a$ then $p^3 \parallel b$.

T F (b) If $p \parallel a$ and $p^2 \parallel (a + b)$ then $p \parallel b$.

T F (c) If x is irrational then \sqrt{x} is irrational.

T F (d) If x and y are irrational then xy is irrational.

T F (e) If $f(n)$ is multiplicative with $f(2) = 3$ and $f(5) = 2$ then $f(20) = 18$.

T F (f) The number of primes from 1 to x denoted $\pi(x)$ satisfies $\lim_{x \rightarrow \infty} \frac{\pi(x)}{x} = 0$.

T F (g) If $\tau(n) = 5$ then $\tau(n^2) = 9$.

T F (h) There is no Pythagorean triple $x^2 + y^2 = z^2$ with $x = 6$.

T F (i) The Fibonacci numbers satisfy $f_{2n} + f_{2(n+1)} = f_{2(n+2)}$.

T F (j) The least residue of $\underbrace{55555555555555555555}_{20 \text{ times}}$ (mod 9) is 5.

T F (k) If $n < 840$ and n is not divisible by any $k \leq 25$ then n is prime.

T F (l) If $p^i \parallel ab$ and $(a, b) = 1$ then $p^i \parallel a$ or $p^i \parallel b$.

9. (8 points) Prove that $\sqrt[4]{\frac{8}{9}}$ is irrational.