Name _____

INTRODUCTION TO NUMBER THEORY

Exam 2

March 17, 2000

The point value of each problem is given in the margin. Standard notation is used:
(a, b) =GCD, $[a, b]$ =LCM, $p^k \| n$ if $p^k | n$ but $p^{k+1} \nmid n$.
$d(n)$ = the number of positive divisors of n.
$\sigma(n)$ = the sum of the positive divisors on n.
$\phi(n)$ = the number of integers relatively prime to n from 1 to n.
$\mu(n) = 1$ if $n = 1$, 0 if $p^2 | n$, and $(-1)^k$ if $n = p_1 \dots p_k$.

(16) 1. Let $a = 2^5 5^8 7^2$, $b = 2^9 3^5 5^7$. Find the following.

(i) $(a, b) =$ The prime factorization will do!

(ii) $[a, b] =$ Same comment.

(iii) The value e such that $2^e \| b^2 a$

(iv) The value f such that $5^f \| (b - a)$.

(12) 2. Use the Sieve of Eratosthenes to find all the primes between 180 and 200. What is the largest prime divisor that must be sifted out?

180	181	182	183	184	185	186	187	188	189
190	191	192	193	194	195	196	197	198	199

Primes:

(12) 3. (a) If $2^k + 1$ is a prime, what can be said about k?

(b) Find a nontrivial factor of $10^9 + 1$. (It doesn't have to be a prime.)

(c) Give a prime divisor of $2^{45} - 1$. (One will do.)

(19) 4. Find the following.

(a) The prime power factorization of 270.

(b) $d(270) =$

(c) $\sigma(270) =$

(d) $\phi(270) =$

(e) Is 270 abundant, deficient or perfect?

(9) 5. Suppose that f is a multiplicative function defined on N such that $f(2) = 3$, $f(3) = 5$, $f(5) = 10$ and $f(p^2) = 0$ for any prime p. For each of the following find the value or state that it cannot be determined based on the given information.

(a) $f(30) =$

(b) $f(50) =$

(c) $f(27) =$

(12) 6. Give a proof that there are infinitely many primes.

(10) 7. Let $n = 2^k(2^{k+1} - 1)$, where k is a positive integer and $2^{k+1} - 1$ is a prime. Prove that n is a perfect number, that is, $\sigma(n) = 2n$.

(10) 8. Let $F(n) = \sum_{d|n} d\mu(d)$.

(a) Show that the function $f(n) = n\mu(n)$ is multiplicative. (You may assume μ is multiplicative.)

(b) Find $F(p^k)$ for any prime power p^k.

(c) Find $F(3000) =$