Name ________________________________

INTRODUCTION TO NUMBER THEORY

Exam 1

February 18, 2000

The point value of each problem is given in the margin.

(10) 1. Find integers q, r such that $-25 = 6q + r$ with

a) $0 \leq r < 6$.

b) $|r| \leq 3$.

(10) 2. Use the Euclidean Algorithm to find the greatest common divisor of 42 and 280.

(14) 3. Find the general solution of the linear equation $46x - 28y = 6$.

(12) 4. Use properties of congruences to compute the least residue of the following numbers modulo 7. (Avoid long multiplication in Z.)

(a) $707 \cdot 145 - 17$

(b) $78^2 + 72^5$

(12) 5. Prove the following theorem. If a, b, c are integers such that $a|bc$ and $(a, b) = 1$ then $a|c$.

(18) 6. True, False. Circle T or F. True means that the statement is true for **all** choices of integers a, b, c, d. (a, b) =GCD. $[a, b]$ =LCM.

T F a) For any integer a, $0|a$.

T F b) If $a|b$ and $a|c$ then $a|(2b - c)$.

T F c) If $a|bc$ then either $a|b$ or $a|c$.

T F d) If $d|a$ and $d|b$ then $d|[a, b]$.

T F e) For any integer q, $(17 - 5q, 5) = 1$

T F f) If $6|(a + b)$ then $a \equiv -b \pmod 6$.

T F g) For any a, the equation $3x - 6y = a$ has a solution in Z.

T F h) Any integer can be expressed as a linear combination of 7 and 11.

T F i) If $f_1 = 1, f_2 = 1, f_3 = 2, \ldots$ is the Fibonacci sequence, then for any positive integer n, $f_n + f_{2n} = f_{3n}$.

(12) 7. Prove by induction that $4^n \equiv 1 + 3n \pmod{9}$, for any positive integer n.

(12) 8. Prove that any positive integer $n > 1$ can be expressed as a product of primes.