Name:

MATH 506 Number Theory – **Exam II**

Wednesday March 26, 2003

Check that that you have all three pages - note that page two is on the back of page one

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1. (12 points) Let $a = 2^3 5^8 13^2$, $b = 2^5 3^7 5^3$. Find the following:

(a) The prime factorization of (a, b)

(b) The prime factorization of $[a, b]$

(c) The value of e if $2^e || a^3 b$

(d) The value of f if $5^f || (a + b)$

2. (10 points) Use induction to prove that for all positive integers n

$$11^n \equiv 10n + 1 \pmod{100}.$$

3. (8 points) Sieve out the primes from 148 to 165. Primes =

148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165

Which prime divisors had to be sifted out?

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4. (22 points) Let p, q denote distinct primes, and recall that

$$\tau(n) = \text{ the number of positive divisors of } n, \quad \sigma(n) = \text{ the sum of the positive divisors of } n.$$

(a) Find the prime factorization of $350 =$

(b) $\tau(350) =$

(c) $\sigma(350) =$

(d) $\tau(p^4 q^2) =$

(e) Give the positive divisors of $p^4 q^2$ (table form is fine).

(f) Describe the prime factorizations possible for n if $\tau(n) = 10$.

(g) What is the smallest positive integer n with $\tau(n) = 10$?

5. (10 points) Prove that any postal amount greater than or equal to 12 cents can be made up using just 3 cent and 7 cent stamps.

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6. (8 points)
(a) Find the missing digit ? that makes 1260054?413000 a multiple of 9.

(b) Find h if $2^h||1305179863251911732518 52$.

7. (20 points) Circle True (T) or False (F).

T F (a) If p is a prime then $p|a^3b^2$ imples that $p|a$ or $p|b$.

T F (b) If x is rational and y is irrational then $(x+y)$ is irrational.

T F (c) If a positive integer $n < 280$ has no divisor $1 < d \leq 14$ then n is prime.

T F (d) The number of primes from 1 to x, denoted $\pi(x)$, satisfies $\lim_{x\to\infty} \frac{\pi(x)}{x \ln x} = 1$.

T F (e) If $f(n)$ is multiplicative then $f(18) = f(3)f(6)$.

T F (f) If $p||a$ and $p||b$ then $p||(a+b)$.

T F (g) The Fibonacci numbers satisfy $f_{3n} = f_{2n} + f_n$.

T F (h) The least residue modulo 11 of $\underbrace{3333333333333333}_{16 \text{ times}}$ is 3.

T F (i) The Fibonacci numbers satisfy $f_{n+2} = 2f_n + f_{n-1}$.

T F (j) If $a = 2^3 5^2 13$ and $b = 2^{11} 5^3$ then $b|a^4$.

8. (10 points) Prove that $\sqrt[3]{18}$ is irrational.

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