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MATH 312 Finite Math – **Final Exam**

Tuesday, December 16, 2003

Check that that you have all six pages - note that the pages are double-sided. Show all your work.

1. (10 points) Use the simplex method to maximize $P = 2x - 3y + z$ subject to the constraints

$$2x - 3y + 3z \leq 12,$$
$$x - 2y + 5z \leq 4$$
$$x \geq 0, \quad y \geq 0, \quad z \geq 0.$$

(a) Introduce slack variables u, v and set up the initial simplex tableau for the problem.

(b) Apply the simplex method. The maximum value is $P =$ ____, achieved when $x =$ ____, $y =$ ____, $z =$ ____, $u =$ ____, $v =$ ____.

2. (12 points) In a game of chance you roll a red and a green die. If the total on the dice is eight you win \$50, if the total is twelve you win \$92.

(a) What is the probability of rolling twelve?

(b) What is the probability of rolling eight?

(c) If it costs \$12 to enter what is your expected gain or loss from playing the game?

PG SCORE

3. (10 points) The mean length of the adult trout in a lake is found to be 20cm with standard deviation 5cm. Assuming a normal distribution, find the probability that a random fish is:

(a) Between 15 and 20cm long.

(b) More than 30cm long.

4. (10 points) (a) Bob pays \$100 a month into an account offering 6% compounded monthly. How much will be in the account after 2 years?

(b) What are the monthly payments on a 10 year mortgage of \$100,000 at an interest rate of 12% compounded monthly.

5. (5 points) An icecream manufacturer asks 100 people to taste their new Apple, Banana and Chocolate flavors. The Apple was liked by 47, the Banana by 49, the Chocolate by 52, Apple & Banana by 33, Apple & Chocolate by 30, Banana & Chocolate by 32, while 18 liked none of them. How many liked all three flavors?

6. (5 points) On a Venn diagram shade the region corresponding to

$$(A \cap B)' \cup C.$$

PG SCORE

7. (12 points) For the matrices

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 3 \\ 3 & -2 \\ 4 & 5 \end{bmatrix}$$

calculate the following or indicate why it is not possible:

(a) AC

(b) CA

(c) Find the inverse matrix A^{-1}.

8. (10 points) Use the Gauss-Jordan method or the formula involving determinants to find the inverse B^{-1} of the matrix

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -4 \\ 1 & 3 & 4 \end{bmatrix}.$$

Show each step and check that $B^{-1}B$ is the identity matrix.

PG SCORE

9. (10 points) Use Gaussian elimination to solve the system of linear equations

$$\begin{aligned} x + 3y + z &= 2 \\ 2x + y - 3z &= 9 \\ 3x + 2y - 3z &= 10. \end{aligned}$$

You will get few points if you don't use the matrix approach. Don't stop before the matrix is in diagonal form even if you see the solution. Check your solution!

10. (13 points) Santa's workshop receives an order for a total of 5000 toy fire trucks, cars & train sets. At most 3000 of the 5000 must be fire trucks and there must be at least as many cars as train sets. If it costs the elves \$6 to produce the fire trucks, \$4 for the cars and \$8 for the train sets, how many fire trucks, cars and train sets should they make to minimize their costs?

$$\begin{aligned} x &= \text{ the number of fire trucks} \\ y &= \text{ the number of cars.} \end{aligned}$$

Your final answers should contain only x and y.

(a) Give the objective function.

(b) Give the constraints for the problem. DO NOT SOLVE IT!

PG SCORE

11. (12 points) Use the simplex method to *minimize* $C = 3x + 4y$ subject to the constraints

$$\begin{aligned} 4x - 2y &\geq -12 \\ -6x + 2y &\geq 10 \\ x \geq 0,\ \ y &\geq 0 \end{aligned}$$

Note that this problem is in non-standard form.
(a) Introduce slack variables u, v, and set up the initial tableau.

b) Apply the simplex method.

(c) The minimum value is $C =$ ____, achieved when $x =$ ____, $y =$ ____, $u =$ ____, $v =$ ____.

12. (16 points) A box contains 10 old batteries. Assume that the probability of a given battery being faulty is 20%. Think of this as 10 Bernoulli trials with random variable X representing the number of faulty batteries.
(a) What is the probability that the box contains exactly three faulty batteries?

(b) What is the probability that the box contains at least one faulty battery?

(c) What is the expected number of faulty batteries in the box?

(d) What is the standard deviation?

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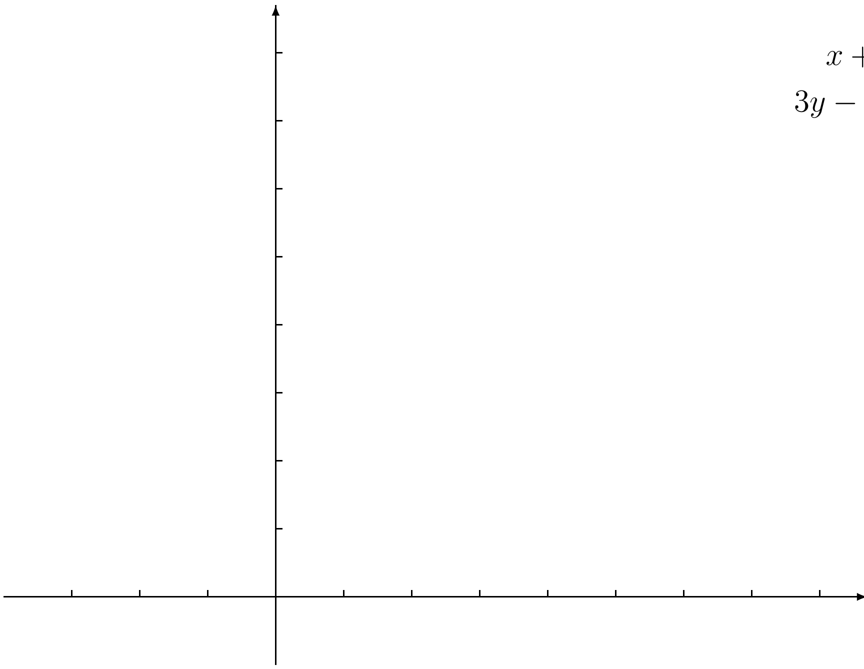
13. (10 points) (a) Put the inequalities in standard form and graph their feasible set. Give the coordinates of the vertices of this region.



$$\begin{aligned} x + y &\leq 7 \\ 3y - 2x &\geq 6 \\ x &\geq 0 \\ y &\geq 0. \end{aligned}$$

(b) What is the maximum value taken by $C = 5x - 2y$ subject to the inequalities in (a).

14. (15 points) A bag contains four fair coins and a coin with heads on both sides. A coin is drawn out of the bag at random and tossed.
(a) Draw a tree diagram for this experiment.

(b) What is the probability that the toss was heads?

(c) If the outcome of the toss was heads, what is the probability that the coin picked was double headed?

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