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MATH 312 Finite Math – Exam II

Wednesday, November 12, 2003

Check that that you have all four pages - note that the pages are double-sided.
Show all your work. If you have sufficient time evaluate factorials and binomial coefficients.

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1. (10 points) Use the simplex method to maximize $P = -5x + 3y + z$ subject to the constraints

$$2x + y - 2z \leq 6,$$
$$x + 2y + z \leq 8$$
$$x \geq 0, \;\; y \geq 0, \;\; z \geq 0.$$

(a) Introduce slack variables u, v and set up the initial simplex tableau for the problem.

(b) Apply the simplex method. The maximum value is $P =$ ____, achieved when $x =$ ____, $y =$ ____, $z =$ ____, $u =$ ____, $v =$ ____.

2. (8 points) Five houses in a row are to be painted with one of the six colors red, green, yellow, purple, white or blue. How many different ways can they be painted so that:
(a) No two adjacent houses are the same color?

(b) No two houses are the same color and the middle house is not red?

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3. (5 points) A coin is flipped five times. What is the probability of getting exactly two heads?

4. (5 points) In the binomial expansion of $(2x - y)^{10}$ what is the coefficient of x^3y^7?

5. (10 points) A group of students take two exams. Suppose that 80% pass the first exam, 60% the second exam. If 50% pass both exams find the probability that:
(a) A student passing the first exam also passes the second.

(b) A student passes at least one of the exams.

(c) Are the events 'passed first exam' and 'passed second exam' independent?

6. (4 points) Suppose that $\mathcal{U} = \{\text{people in Manhattan}\}$ and

$$A = \{\text{men}\}, \quad B = \{\text{married people}\}, \quad C = \{\text{people who like chocolate}\}.$$

Describe the set $(A \cup B)' \cap C$ in words.

7. (5 points) You are dealt 4 cards from a standard pack of 52 cards. How many different hands consist of two aces and two picture cards?

8. (4 points) Twelve professors have to pick a four person library committee and a three person hiring committee. Assuming no one has to serve on both committees how many ways can they do this?

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9. (12 points) Use the simplex method to *minimize* $C = -2x + 3y$ subject to the constraints

$$\begin{aligned} 4x + 2y &\leq 60 \\ -x + y &\geq 10 \\ x \geq 0,\ \ y &\geq 0 \end{aligned}$$

Note that this problem is in non-standard form.
(a) Introduce slack variables u, v, and set up the initial tableau.

b) Apply the simplex method.

(c) The minimum value is $C =$ ____, achieved when $x =$ ____, $y =$ ____, $u =$ ____, $v =$ ____.

10. (6 points) The linear programming problem:
Maximize $P = 3x + 4y + z$ *subject to the constraints*

$$x - y + 2z \leq 11, \quad x + 2y + 3z \leq 14, \quad x \geq 0,\ y \geq 0,\ z \geq 0,$$

has initial and final simplex tableaux

$$\left[\begin{array}{cccccc|c} 1 & -1 & 2 & 1 & 0 & 0 & 11 \\ 1 & 2 & 3 & 0 & 1 & 0 & 14 \\ \hline -3 & -4 & -1 & 0 & 0 & 1 & 0 \end{array}\right] \qquad \left[\begin{array}{cccccc|c} 1 & 0 & 7/3 & 2/3 & 1/3 & 0 & 12 \\ 0 & 1 & 1/3 & -1/3 & 1/3 & 0 & 1 \\ \hline 0 & 0 & 22/3 & 2/3 & 7/3 & 1 & 40 \end{array}\right]$$

Suppose that the second inequality is replaced by $x + 2y + 3z \leq 20$. Use marginal analysis to find the last column of the new initial and final tableaux. The new maximum value is $P =$ ____, achieved when $x =$ ____, $y =$ ____, $z =$ ____, $u =$ ____, $v =$ ____.

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11. (4 points) On a Venn diagram shade the region corresponding to

$$(A \cup B)' \cap C.$$

12. (10 points) 100 people are asked to locate Afghanistan, Iran & Pakistan on a map. Suppose that Afghanistan was corrected identified by 75, Iran by 69, Pakistan by 72, Afghanistan and Iran by 55, Afghanistan and Pakistan by 57, Iran and Pakistan by 56, while 4 failed to identify any of them.
(a) How many identified all three countries?

(b) Draw a final Venn diagram to display this data.

13. (5 points) You plant 4 hyacinth bulbs in a pot and give them as a gift. Suppose the probability of a given bulb failing to grow is 20%. Find the probability that at least one of the bulbs fails to grow.

14. (12 points) A bag contains three fair coins and a coin weighted so that heads occurs 4/5 of the time. A coin is drawn out of the bag at random and tossed.
(a) Draw a tree diagram for this experiment.

(b) What is the probability that the toss was heads?

(c) If the outcome of the toss was heads, what is the probability that the coin picked was weighted?

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