

Title: Maximizing A Class of Functionals over Hyperbolically Convex Functions.

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Abstract: In this talk, we use a method of Barnard, based on Julia Variations to solve an extremal problem for hyperbolically convex functions. A hyperbolically convex function can be viewed as a univalent analytic function sending the unit disk \mathbb{D} of the complex plane \mathbb{C} in to a hyperbolically convex subset of \mathbb{D} . A subset of \mathbb{D} , call it B , is hyperbolically convex provided the hyperbolic geodesic connecting any two distinct points in B lies entirely in B . We denote by H the class of hyperbolically convex functions normalized by fixing the origin. The extremal problem we solve is maximizing Lf over the class H . L is a functional of the form $\Re \left(\Phi \left(\log \frac{f'(z)}{f'(0)} \right) \right)$, with reasonable conditions on Φ . This class includes, by choosing $\Phi = \exp(z)$, the functional $|f'(z)|/|f'(0)|$. As a result, we achieve a broad generalization of a recent result of Pommerenke, Mejía, and Vasiliev, using an entirely different technique. This technique also shows promise in addressing many problems, including a related open conjecture by Pommerenke *et al.*