New Function Spaces that Facilitate Understanding Nonlinear PDEs

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Prominent advances in harmonic analysis and nonlinear partial differential equations will continue to facilitate and enhance several fields of mathematics: geometric function theory, calculus of variation, nonlinear elasticity, material science and so forth. In this process there is a challenge and opportunity for contributions from new function spaces. Among them we find:

- Orlicz spaces $\mathbf{L}^{\Phi}(\Omega)$
- Grand spaces $\mathbf{BL}^{p}(\Omega)$ and $\mathbf{VL}^{p}(\Omega)$
- The corresponding Sobolev classes of weakly differentiable functions.

They arise naturally as domains of definition of basic nonlinear differential operators, like Jacobian determinants and somewhat more general differential expressions called null-Lagrangians, degenerate elliptic equations and mappings of finite distortion. This latter topic reflects a very productive study (over the past decade) of nonlinear PDEs, at the point when the uniform ellipticity is lost. There are also related questions about smooth approximations of weakly differentiable mappings between Riemannian manifolds.

Hopefully this lecture, which I address to a general audience, will give some appreciation of new function spaces, as well as a glimpse of the diversity of the directions in which current nonlinear geometric analysis is moving.

The spaces we need most are
spaces we haven't discovered yet.