

NAME KEY

Final – Part 1 – Friday, May 6, 2005.
Math 320: Math for Elementary School Teachers
20-MINUTES MENTAL DRILL

INSTRUCTIONS: ONLY WRITE THE FINAL ANSWER, NO SCRATCH PAPER, NO SCRIBBLING ON THIS SHEET, NO CALCULATORS, USE INK PEN ONLY. EXERCISE THE COMPUTATIONAL TRICKS (THINKING STRATEGIES) WE HAVE BEEN LEARNING. EACH QUESTION IS WORTH 2 POINT.

1. $(13)_{10} = (1101)_2$

2. $1300 \div 25 =$

52

3. $39^2 =$

1521

4. $99 \times 101 =$

9999

5. $\text{GCF}(645, 700) =$

5

6. Factor $6^2 \cdot 10 =$

$2^3 \cdot 3^2 \cdot 5$

7. $(3 - x) : x = 20 : 10,$

$x = 1$

8. $(\frac{5}{2} - \frac{2}{5}) \div \frac{1}{10} =$

21

9. $\frac{8}{160} = 5\%$ 10. Percentage decrease from 4500 to 3600 =

20%

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FINAL – Part 2 – Friday, May 6, 2005.
Math 320: Math for Elementary School Teachers

PART 2: 30-MINUTES TEST

INSTRUCTIONS: USE SCRATCH PAPER, WRITE COMPLETE AND FINAL ANSWERS USING INK PEN, NO SCRIBBLING, NO CALCULATORS. PARTIAL CREDIT WILL BE GIVEN IF DESERVED, SO JUSTIFY AND SHOW ALL YOUR WORK.

1. (5pts) Fill in the blanks:

(a) "6 is $\frac{3}{4}$ of what?" is the PARTITIVE interpretive question for

$6 \div \frac{3}{4}$

(b) After estimating the quotient and calculating the remainder in long

division, it is necessary to check $0 \leq \text{remainder} < \underline{\text{DIVISOR}}$.

(c) Fundamental Theorem of Arithmetics says that every whole number $N > 1$

IS A PRODUCT OF PRIMES IN A UNIQUE WAY

(d) A proportion is a statement that TWO RATIOS ARE EQUAL

(e) *Division* is defined by THE MISSING FACTOR IN MULTIPLICATION

as in the following example:

$21 \div 3$ IS THE MISSING FACTOR IN $3 \times \underline{\quad} = 21$

2. Teaching problems.

a. (3pts) State the Quotient-Remainder Theorem and explain it using a set model.

FOR ANY TWO WHOLE NUMBERS A AND k WITH $k \neq 0$
THERE ARE UNIQUE WHOLE NUMBERS q AND r SUCH THAT

$$A = (k \times q) + r \quad \text{AND} \quad 0 \leq r < k$$

Ex! $k = 12 = \#$ EGGS IN A CARTON

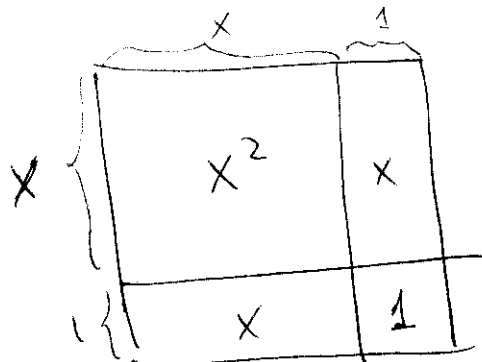
$A = 40 =$ TOTAL # OF EGGS.

3 CARTONS $= 3 \times k = 36$ EGGS AND $4 (< 12)$ ARE LEFT OVER

$$\text{So } 40 = 3 \times 12 + 4.$$

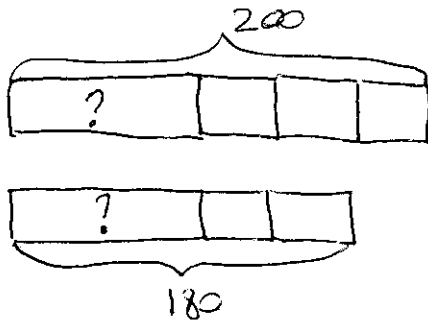
b. (3pts) Explain the identity $(x+1)^2 = x^2 + 2x + 1$ using the distributive property and a diagram.

$$\begin{aligned} (x+1)^2 &= (x+1)(x+1) \\ &= x(x+1) + 1(x+1) \quad \text{DISTR. PROP.} \\ &= x^2 + x + x + 1 \quad \text{" \# ANY-ORDER} \\ &= x^2 + 2x + 1. \quad \text{"} \end{aligned}$$



3. Give a "teacher's solution" for the following word problem. "John had \$200 and David had \$180. After they each spent an equal amount of money, the ratio of John's money to David's money was 3:2. How much did each of them spend?"

a. (2pts) With a bar diagram:



$$1 \text{ UNIT} = 200 - 180 = 20 \$$$

$$2 \text{ UNITS} = 40 \$$$

$$180 - 40 = 140$$

THEY EACH SPENT 140 \$

b. (2pts) With algebra (Hint: let x be the amount they each spent and come-up with a proportion first, then an equation involving x . Then solve the equation for x .)

$$(200 - x) : 3 = (180 - x) : 2$$

$$\Rightarrow \frac{200 - x}{3} = \frac{180 - x}{2}$$

$$\Rightarrow 2(200 - x) = 3(180 - x)$$

$$\Rightarrow 400 - 2x = 540 - 3x$$

$$\Rightarrow 3x - 2x = 540 - 400$$

$$\Rightarrow \boxed{x = 140}$$

4. (4pts) Simplify as much as possible and write the final answer as a product of primes, or powers of primes. Write out every step neatly.

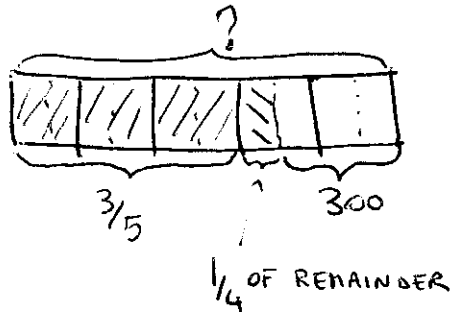
$$\frac{5^3 \cdot 24^2 \cdot 10^0}{8 \cdot 15^2 \cdot 3} =$$

$$= \frac{5^3 \cdot (3 \cdot 8)^2 \cdot 1}{8 \cdot (3 \cdot 5)^2 \cdot 3} = \frac{5^3 \cdot \cancel{3}^2 \cdot 8^2}{\cancel{8} \cdot \cancel{3}^2 \cdot \cancel{5}^2 \cdot 3}$$

$$= \frac{5 \cdot 8}{3} = \boxed{5 \cdot 2^3 \cdot 3^{-1}}$$

5. Give a "teacher's solution" for the following word problem. "Lindsey made some tarts. She sold $\frac{3}{5}$ of them in the morning and $\frac{1}{4}$ of the remainder in the afternoon. If she had 300 tarts left, how many tarts did she make?"

a. (3pts) With a bar diagram:



$$\begin{aligned}
 3 \text{ UNITS} &= 300 \text{ TARTS} \\
 1 \text{ UNIT} &= 100 \quad \text{"} \\
 \Rightarrow 10 \text{ UNITS} &= 1000 \quad \text{"}
 \end{aligned}$$

SHE MADE 1000 TARTS

b. (3pts) With algebra (Hint: let x be "the amount of tarts she made" and come-up with an equation involving x . Then solve the equation for x .)

$$\cancel{x} - \frac{3}{5}x - \frac{1}{4}\left(x - \frac{3}{5}x\right) = 300$$

\uparrow SOLD IN MORNING \uparrow A FOURTH OF THE REMAINDER SOLD IN THE AFTERNOON

$$\Rightarrow x - \frac{3}{5}x - \frac{1}{4} \cdot \frac{2}{5}x = 300$$

$$\Rightarrow x \left(\frac{10 - 6 - 1}{10} \right) = 300$$

$$\Rightarrow x \cdot \frac{3}{10} = 300 \Rightarrow 3x = 3000 \Rightarrow \boxed{x = 1000}$$

6. (5 pts) Find $x > 0$ as determined by the proportion below. Write all the steps you have taken and show the strategies you have used.

$$\left[\frac{4}{5} + \left(3 - \frac{3}{5} \right) \div 2 \right] : x = x : \left(1 - \frac{1}{5} - \frac{3}{10} \right)$$

$$a) \quad 3 - \frac{3}{5} = \frac{15}{5} - \frac{3}{5} = \frac{12}{5} \rightarrow \frac{12}{5} \div 2 = \frac{12}{5} \cdot \frac{1}{2} = \frac{6}{5}$$

$$\frac{4}{5} + \frac{6}{5} = \frac{10}{5} = 2$$

$$b) \quad 1 - \frac{1}{5} - \frac{3}{10} = \frac{10 - 2 - 3}{10} = \frac{5}{10} = \frac{1}{2}$$

$$\text{So} \quad 2 : x = x : \frac{1}{2}$$

$$\frac{2}{x} = \frac{x}{\frac{1}{2}} = 2x$$

$$2 = 2x^2$$

$$x^2 = 1$$

$$\Rightarrow \boxed{x=1}$$