NAME KEY

Exam 3 – Part 1 – Wednesday, Apr. 6, 2005.
Math 320: Math for Elementary School Teachers
20–MINUTES MENTAL DRILL

INSTRUCTIONS: ONLY WRITE THE FINAL ANSWER, NO SCRATCH PAPER, NO SCRIBBLING ON THIS SHEET, NO CALCULATORS, USE INK PEN ONLY. EXERCISE THE COMPUTATIONAL TRICKS (THINKING STRATEGIES) WE HAVE BEEN LEARNING. EACH QUESTION IS WORTH 2 POINTS.

1. Is 2838 a multiple of 4? YES (NO)

2. Factor $140 =$ $2^2 \cdot 5 \cdot 7$

3. $\frac{1-\frac{1}{3}}{1+\frac{1}{2}} =$ $4/9$

4. Find N where $57 < N < \frac{3845}{53}$ $\frac{3600}{60} = 60$

5. $\text{GCF}(110, 130) =$ 10

6. Average $\frac{2}{5}, \frac{3}{10}, \frac{6}{15}$ $\frac{11}{30}$

7. $\text{LCM}(2 \cdot 3 \cdot 7, 5) =$ 210

8. $10 \cdot \frac{2}{3} + 5 \cdot \frac{5}{3} =$ 15

9. 44 times "24 and 1 eleventh" = 1060

10. Three fourths divided by four fifths = $\frac{15}{16}$

NAME KEY

EXAM 3 – Part 2 – Wednesday, Apr. 6, 2005.
Math 320: Math for Elementary School Teachers

PART 2: 30–MINUTES TEST

INSTRUCTIONS: USE SCRATCH PAPER, WRITE COMPLETE AND FINAL ANSWERS USING INK PEN, NO SCRIBBLING, NO CALCULATORS. PARTIAL CREDIT WILL BE GIVEN IF DESERVED, SO JUSTIFY AND SHOW ALL YOUR WORK.

1. (5pts) Fill in the blanks:

(a) An even number is TWICE A WHOLE NUMBER

and an odd number is ONE MORE THAN TWICE A WHOLE NUMBER

(b) A is divisible by k whenever A is a MULTIPLE of k,

or k is a FACTOR of A.

(c) Fundamental Theorem of Arithmetics: Every whole number $N > 1$

IS A PRODUCT OF PRIMES IN A UNIQUE WAY

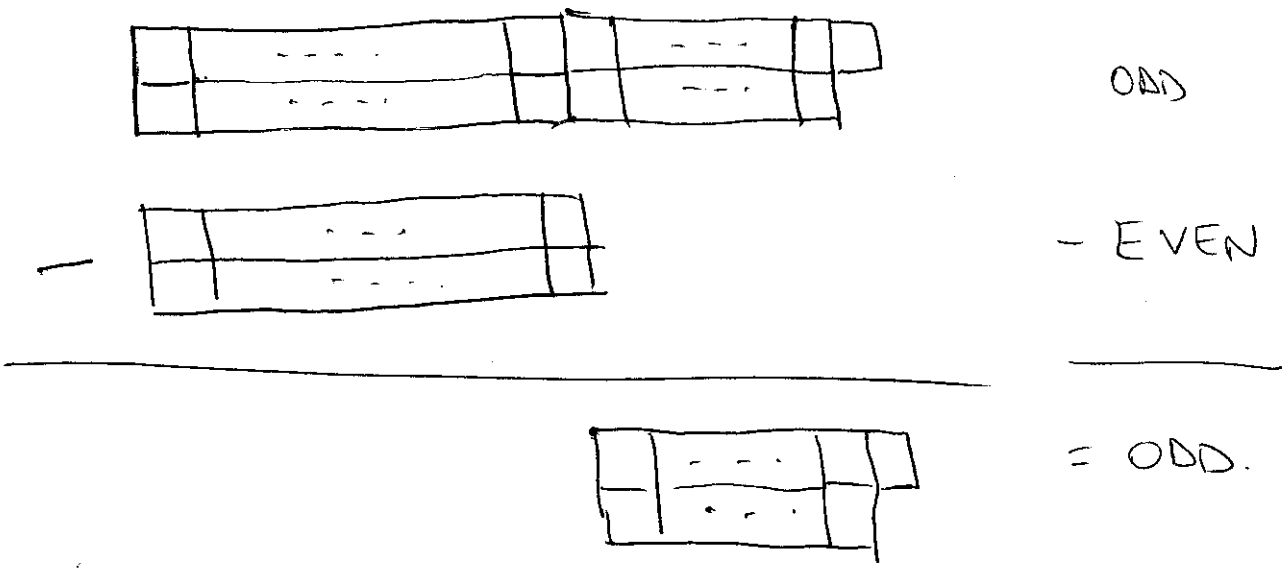
(d) The numerator of a fraction is the number of FRACTIONAL

UNITS.

(e) Give the interpretive question for $1\,3/4 \div \frac{1}{2}$ in partitive division:

1 3/4 IS 1/2 OF WHAT ?

2. **(5pts)** Give a "picture proof" and an "algebra proof" of the fact that if you subtract a smaller even number from an odd number the result is still odd.



REPRESENT THE ODD N.° BY $2K+1$

AND THE EVEN ONE BY $2L$

THEN: $(2K+1)-2L = 2K+1-2L$

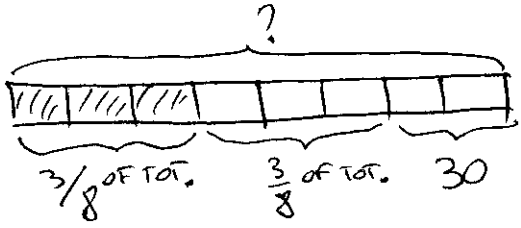
$= 2(K-L)+1$

(HERE $K-L \geq 0$ BECAUSE $2(K-L)+1$ IS.)

THE RESULT IS STILL ODD.

3. (6pts) Give a full "teacher's solution" for the following two word problems. Use both a bar diagram and pre-algebra, and finally show the arithmetic calculations.

"$\frac{3}{8}$ of a group of children are boys. There are 30 more girls than boys. How many children are there altogether?"



2 UNITS = 30 CHILDREN

8 UNITS = 120 CHILDREN

THERE ARE 120 CHILDREN ALTOGETHER

X = # OF CHILDREN

OF BOYS = $\frac{3}{8}X$

OF GIRLS = $\frac{3}{8}X + 30$

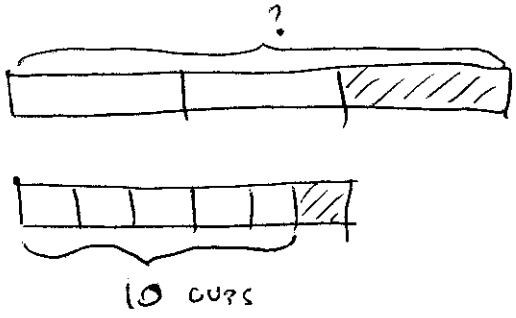
SO: $\frac{3}{8}X + \frac{3}{8}X + 30 = X$

$\Rightarrow \frac{6}{8}X + 30 = X$

$\Rightarrow 30 = X - \frac{3}{4}X = \frac{1}{4}X$

$\Rightarrow X = 4 \cdot 30 = 120$

"Jane used $\frac{1}{3}$ of the flour to bake some cookies and $\frac{1}{6}$ of the rest to bake some cake. If she has 10 cups of flour left, how much flour did she start with?"



5 UNITS = 10 CUPS

1 UNIT = 2 CUPS

9 UNITS = 18 CUPS

SHE STARTED WITH 18 CUPS

X = # CUPS AT BEGINNING

CUPS AFTER COOKIES = $X - \frac{1}{3}X = \frac{2}{3}X$

CUPS AFTER CAKE = $\left(\frac{2}{3}X\right) - \frac{1}{6}\left(\frac{2}{3}X\right)$

SO $10 = \left(1 - \frac{1}{6}\right)\frac{2}{3}X = \frac{5}{6} \cdot \frac{2}{3}X$

$\Rightarrow 10 = \frac{5}{9}X$

SO $X = 10 \div \frac{5}{9} = 10 \times \frac{9}{5}$

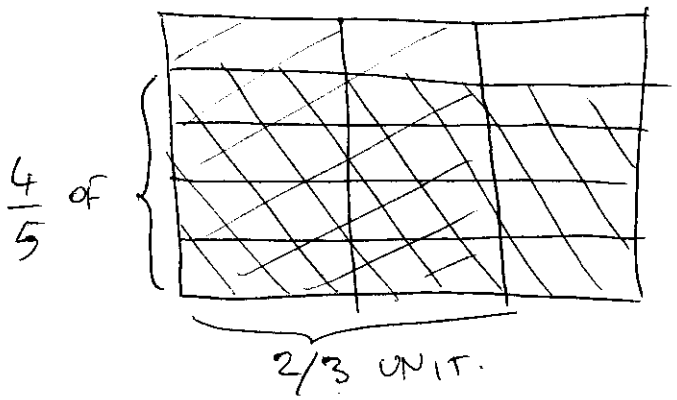
$= \frac{10}{5} \times 9 = 2 \times 9 = 18$

4. (3pts) Give an algebra proof of the fact that a 3-digit number $(abc)_{10}$ is divisible by 9 if $a+b+c$ is. Hint: use the place value expansion of $(abc)_{10}$.

$$(abc)_{10} = 100a + 10b + c = (99+1)a + (9+1)b + c$$
$$= 99a + a + 9b + b + c$$
$$= 9 \times 11a + 9b + (a+b+c)$$
$$= \underbrace{9(11a+b)}_{\text{MULTIPLE OF 9}} + (a+b+c)$$

SO $(abc)_{10}$ IS DIVISIBLE BY 9 IF AND ONLY IF $(a+b+c)$ IS _____

5. (2pts) Illustrate how to multiply $\frac{4}{5} \times \frac{2}{3}$ using an area model.



$$\frac{4}{5} \times \frac{2}{3} = \frac{4}{5} \text{ of } \frac{2}{3} = \frac{8}{15}$$

6. **(4pts)** Estimate the square root of 143 and use the primality test to check if 143 is prime.

$11^2 = 121 < 143 < 144 = 12^2$

SO WE NEED ONLY CHECK DIVISIBILITY BY

2, 3, 5, 7, 11

IS 143 DIVISIBLE BY 2 ? NO.

BY 3 ? NO (1+4+3=8)

BY 5 ? NO

BY 7 ? NO (140 ÷ 7 = 20)

BY 11 ? YES!

$$11 \overline{)143} \quad 13$$
$$\underline{11}$$
$$33$$

$143 = 11 \times 13$ IS COMPOSITE.

7. (5 pts) Compute the expression below using the arithmetic properties to your advantage. Write all the steps you have taken and show the strategies you have used.

$$\left(\frac{1}{4}-\frac{9}{25}\times\frac{5}{18}\right)^2\div\left(1+\frac{1}{2}\right)^2-\left(\frac{1}{10}\right)^2=$$

$$=\left(\frac{1}{4}-\frac{1}{10}\right)^2\div\left(\frac{3}{2}\right)^2-\left(\frac{1}{10}\right)^2$$

$$=\left(\frac{6}{40}\right)^2\times\left(\frac{2}{3}\right)^2-\left(\frac{1}{10}\right)^2$$

$$=\frac{\cancel{3^2}}{20^2}\times\frac{2^2}{\cancel{3^2}}-\frac{1}{10^2}$$

$$=\frac{1}{\cancel{2^2}\cdot 10^2}\times\cancel{2^2}-\frac{1}{10^2}$$

$$=\frac{1}{10^2}-\frac{1}{10^2}=\boxed{0}$$