The State of State MATH Standards

2005

by David Klein

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Statewide academic standards are important, not only because they provide goal posts for teaching and learning, but also because they drive education policies. Standards determine—or should determine—the content and emphasis of tests used to measure student achievement; they influence the selection of textbooks; and they form the core of teacher education programs. The quality of a state's K-12 academic standards has far-reaching consequences for the education of its citizens.

The quality of state mathematics standards was the subject of two previous reports from the Thomas B. Fordham Foundation, both authored by Ralph Raimi and Lawrence Braden. The first, published in March 1998 (which we refer to as Fordham I), was a pioneering work. Departing from previous such undertakings, it exposed the shocking inability of most state education bureaucracies even to describe what public schools should teach students in math classes. The average national grade was a D. Only three states received A grades, and more than half received grades of D or F. "On the whole," wrote the authors in 1998, "the nation flunks."

The Fordham I grades were based on numerical scores in four categories: clarity, content, reasoning, and negative qualities. Using these same criteria, the Foundation released Raimi and Braden's second report in January 2000 (which we refer to as Fordham II). It evaluated 34 new or revised state documents and retained the original evaluations of 15 states whose math standards had not changed since Fordham I. The result was a national average grade of C, an apparent improvement. However, Fordham II, like Fordham I, cautioned readers not to take the overall average grade as a definitive description of performance, and to read the scores (0 to 4 possible points) for the four criteria separately, to arrive at an understanding of the result. Ralph Raimi made clear in his introduction to Fordham II that much of the increase of the final grades was due to improved clarity. States had improved upon prose that Raimi termed "appallingly vague, so general as to be unusable for guiding statewide testing or the choice of textbooks." The result was that many states had by the time of Fordham II achieved higher overall grades through little more than a clearer exposition of standards with defective mathematical content.

Major Findings

The criteria for evaluation used in this report are the same as in Fordham I and II. For the reader's convenience, these criteria are defined and described in the next section. *However, this report differs from Fordham I and II in that the relative weights of the criteria have been changed*. At the suggestion of Raimi and Braden, we increased the weight of the content criterion and reduced uniformly the weights of the other three criteria: clarity, reason, and negative qualities. Content now accounts for 40 percent of a state's total score, compared to 25 percent in Fordham I and II. This affects the final numerical scores upon which our grades are based and, in some cases, results in lower grades, especially for states that benefited from higher "clarity" scores in Fordham II. The individual state reports beginning on page 37 include numerical scores for each criterion. The Appendix, on page 123, also includes numerical scores for subcategories of these four criteria.

The consensus of the evaluating panel of mathematicians is that this weighting properly reflects what is most important in K-12 standards in 2005. Content is what matters most in state standards; clear but insubstantial expectations are insufficient.

With the greater weight attached to mathematical content in this report, it is not surprising that our grades are lower than those of Fordham II. In fact, our grade distribution more closely resembles that of Fordham I. We assigned A, or "excellent," grades to only three states: California, Indiana, and Massachusetts. The national average grade is D, or "poor," with most states receiving D or F grades. The table below shows the scores and grade assignments for 49 states and the District of

Columbia (which for purposes of this report we refer to as a state). Only Iowa is missing, because it has no standards documents.

Besides the different weighting of criteria for evaluation, another caveat for those wanting to compare Fordham I and II with this report to identify trends over time is the change of authorship. None of the mathematicians who scored and evaluated the state math standards in 2005 had any involvement in Fordham I and II. However, Ralph Raimi and Lawrence Braden served as advisers for this project, and helped to resolve many technical questions that arose in the course of evaluating state documents. We describe this interaction in greater detail in the section, "Methods and Procedures," on page 121.

Common Problems

What are some of the reasons that so many state mathematics standards come up short? We discuss here nine problems found in many, and in some cases most, of the standards documents that we reviewed.

Calculators

One of the most debilitating trends in current state math standards is overemphasis of calculators. The majority of state standards documents call upon students to use calculators starting in the elementary grades, often beginning in Kindergarten and sometimes even in pre-Kindergarten. For example, the District of Columbia requires that the pre-Kindergarten student "demonstrates familiarity with basic calculator keys." New Hampshire directs Kindergarten teachers to "allow students to explore one-more-than and one-less-than patterns with a calculator" and first grade teachers "have students use calculators to explore the operation of addition and subtraction," along with much else. In Georgia, first-graders "determine the most efficient way to solve a problem (mentally, paper/pencil, or calculator)." According to New Jersey's policy:

> *Calculators can and should be used at all grade levels to enhance student understanding of mathematical concepts. The majority of questions on New Jersey's*

Fig. 3: State Grades, Alphabetical Order

STATE	Clarity	Content	Reason	Negative Qualities	Final G.P.A.	2005 GRADE
AL	3.00	3.17	2.00	3.50	2.97	B
AK	2.00	1.17	0.50	1.75	1.32	D
AZ	2.00	2.00	2.00	2.00	2.00	C
AR	1.50	0.67	0.00	0.75	0.72	F
CA	3.83	3.94	3.83	3.92	3.89	A
CO	1.00	1.67	1.00	1.50	1.37	D
CT	0.67	0.33	0.00	1.00	0.47	F
DE	0.83	0.67	0.50	0.00	0.54	F
DC	1.67	1.33	1.50	1.00	1.37	D
FL	1.33	0.67	1.50	0.50	0.93	F
GA	3.33	2.67	2.00	2.00	2.53	B
HI	1.00	0.33	0.00	0.50	0.43	F
ID	1.67	0.67	1.00	1.50	1.10	D
IL	1.50	2.00	1.00	2.50	1.80	C
IN	3.67	3.83	4.00	3.75	3.82	A
IA	-	-	-	-	-	-
KS	1.67	0.94	0.33	0.25	0.83	F
KY	1.83	2.33	1.00	1.50	1.80	C
LA	2.00	2.33	1.00	1.25	1.78	C
ME	1.17	1.17	0.50	2.75	1.35	D
MD	2.00	1.67	1.50	2.00	1.77	C
MA	3.67	3.67	2.00	3.50	3.30	A
MI	2.17	1.67	2.00	2.50	2.00	C
MN	2.00	1.67	1.00	2.00	1.67	D
MS	1.33	2.00	1.00	2.00	1.67	D
MO	0.67	0.33	1.00	0.50	0.57	F
MT	1.00	1.00	0.00	2.00	1.00	D
NE	1.72	1.28	0.67	2.17	1.42	D
NV	2.17	1.33	1.50	2.50	1.77	C
NH	1.17	0.67	0.00	1.00	0.70	F
NJ	2.17	1.17	0.50	0.75	1.15	D
NM	3.00	2.67	2.00	3.00	2.67	B
NY	1.50	2.33	2.00	2.25	2.08	C
NC	2.33	1.50	1.50	2.25	1.82	C
ND	2.33	1.33	1.00	3.00	1.80	C
OH	2.00	1.33	1.00	1.50	1.43	D
OK	2.17	1.83	1.50	2.50	1.97	C
OR	2.50	1.00	0.00	2.25	1.35	D
PA	1.33	1.17	1.00	1.75	1.28	D
RI	1.00	0.67	0.00	1.00	0.67	F
SC	1.00	1.67	1.50	0.75	1.32	D
SD	2.17	1.67	1.00	2.50	1.80	C
TN	1.83	1.33	2.00	2.00	1.70	D
TX	2.67	1.67	1.00	2.00	1.80	C
UT	1.83	1.17	0.50	1.00	1.13	D
VT	1.33	1.00	0.67	2.00	1.20	D
VA	2.83	2.00	1.50	1.50	1.97	C
WA	0.33	1.00	0.50	0.00	0.57	F
WV	2.00	2.50	3.00	1.75	2.35	C
WI	1.67	1.67	1.00	1.50	1.50	D
WY	1.00	0.83	0.00	2.25	0.98	F
Average	1.85	1.57	1.15	1.79	1.59	D

(A = 4.00 - 3.25; B = 3.24 - 2.50; C = 2.49 - 1.75; D = 1.74 - 1.00; F = 0.99 - 0.00) *new third- and fourth-grade assessments in mathematics will assume student access to at least a four-function calculator.*

Alaska's standards explicitly call upon third-graders to determine answers "to real-life situations, paper/pencil computations, or calculator results by finding 'how many' or 'how much' to 50." For references and a nearly endless supply of examples, we refer the reader to the state reports that follow.

Calculators enable students to do arithmetic quickly, without thinking about the numbers involved in a calculation. For this reason, using calculators in a high school science class, for example, is perfectly sensible. There, the speed and efficiency of a calculator keep the focus where it belongs, on science, much as the slide rule did in an earlier era. At that level, laborious hand calculations have no educational value, because high school science students already know arithmetic—or they should.

By contrast, elementary school students are still learning arithmetic. The main goal of elementary school mathematics education is to get students to think about numbers and to learn arithmetic. Calculators defeat that purpose. They allow students to arrive at answers without thinking. Hand calculations and mental mathematics, on the other hand, force students to develop an intuitive understanding of place value in the decimal system, and of fractions. Consider the awkwardly written Alaska standard cited above. Allowing third-graders to use calculators to find sums to 50 is not only devoid of educational value, it is a barrier to sound mathematics education. Some state standards even call for the use of fraction calculators in elementary or middle school, potentially compromising facility in rational number arithmetic, an essential prerequisite for high school algebra.

An implicit assumption of most state standards is that students need practice using calculators over a period of years, starting at an early age. Thus, very young children are exposed to these machines in order to achieve familiarity and eventual competence in their use. But anyone can rapidly learn to press the necessary buttons on a calculator. Standards addressing "calculator skills" have no more place in elementary grade standards than do standards addressing skills for dialing telephone numbers.

With proper restriction and guidance, calculators can play a positive role in school mathematics, but such direction is almost always missing in state standards documents. A rare exception is the California *Framework*, which warns against over-use, but also identifies specific topics, such as compound interest, for which the calculator is appropriate. As in many European and Asian countries, the California curriculum does not include calculators for any purpose until the sixth grade, and thereafter only with prudence.

Many states diminish the quality of their standards by overemphasis of calculators and other technology, not only in the lower grades, but even at the high school level. Standards calling for students to use graphing calculators to plot straight lines are not uncommon. Students should become skilled in graphing linear functions by hand, and be cognizant of the fact that only two points are needed to determine the entire graph of a line. This fundamental fact is easily camouflaged by the obsessive use of graphing technology. Similarly, the use of graphing calculators to plot conic sections can easily and destructively supplant a mathematical idea of central importance for this topic and others: completing the square.

Memorization of the Basic Number Facts

We use the term "basic number facts" to refer to the sums and products of single-digit numbers and to the equivalent subtraction and division facts. Students need to memorize the basic number facts because doing so frees up working memory required to master the arithmetic algorithms and tackle applications of mathematics. Research in cognitive psychology points to the value of automatic recall of the basic facts.[1] Students who do not memorize the basic number facts will founder as more complex operations are required of them, and their progress in mathematics will likely grind to a halt by the end of elementary school.

[1] A cogent summary of some of that research appears on pages 150-151 and 224 of *The Schools We Need: And Why We Don't Have Them*, by E.D. Hirsch, Jr., Doubleday, 1996.

Unfortunately, many states do not explicitly require students to memorize the basic number facts. For example, rather than memorizing the addition and subtraction facts, Utah's second-graders "compute accurately with basic number combinations for addition and subtraction facts to eighteen," and, rather than memorize the multiplication and division facts, Oregon's fourth-graders are only required to "apply with fluency efficient strategies for determining multiplication and division facts 0-9." Computing accurately that $6 + 7 = 13$ and using efficient strategies to calculate that $6 \times 7 = 42$ is not the same as memorizing these facts. We are not suggesting that the meaning of the facts should not also be taught. Students should of course understand the meaning of the four arithmetic operations, as well as ways in which the basic number facts can be recovered without memory. All are important. But there is no *real* fluency without memorization of the most basic facts. The states that decline to require this do their students a disservice.

The Standard Algorithms

Only a minority of states explicitly require knowledge of the standard algorithms of arithmetic for addition, subtraction, multiplication, and division. Instead, many states do not identify any methods for arithmetic, or worse, ask students to invent their own algorithms or rely on *ad hoc* methods. One of Connecticut's standards documents advises,

> *Instructional activities and opportunities need to focus on developing an understanding of mathematics as opposed to the memorization of rules and mechanical application of algorithms.*

This is insufficient. Specialized methods for mental math work well in some cases but not in others, and it is unwise for schools to leave students with untested, private algorithms for arithmetic operations. Such procedures might be valid only for a subclass of problems. The standard algorithms are powerful theorems and they are *standard* for a good reason: they are guaranteed to work for all problems of the type for which they were designed.

Knowing the standard algorithms, in the sense of being able to use them and understanding how and why they work, is the most sophisticated mathematics that an elementary school student is likely to grasp. Students who have mastered these algorithms gain confidence in their ability to compute. They know that they can solve any addition, subtraction, multiplication, or division problem without relying on a mysterious black box, such as a calculator. Moreover, the ability to execute the arithmetic operations in a routine manner helps students to think more conceptually. As their use of the standard algorithms becomes increasingly automatic, students come to view expressions such as $6485 - 3689$ as a single number that can be found easily, rather than thinking of it as a complicated problem in itself. If mathematical thinking is the goal, the standard algorithms are a valuable part of the curriculum.

A wide variety of algorithms are used in mathematics and engineering, and our technological age surrounds us with machines that depend on the algorithms programmed into them. Students who are adept with the most important and fundamental examples of algorithms—the standard algorithms of arithmetic—are well positioned to understand the meaning and uses of other algorithms in later years.

One benefit of learning the long division algorithm is that it requires estimation of quotients at each stage. If the next digit placed in the (trial) answer is too large or too small, that stage has to be done over again, and the error is made visible by the procedure. Number sense and estimation skills are reinforced in this way. The long division algorithm illustrates an important idea in mathematics: repeated estimations leading to increasingly accurate approximations.

The long division algorithm has applications that go far beyond elementary school arithmetic. At the middle school level, it can be used to explain why rational numbers have repeating decimals. This leads to an understanding of irrational, and therefore real numbers. Division is also central to the Euclidean Algorithm for the calculation of the greatest common divisor of two integers. In high school algebra, the long division algorithm, in slightly modified form, is used for division of polynomials. At the university level, the algorithm is generalized to accommodate division of power series and it is also important in advanced abstract algebra. Experience with the long division algorithm in elementary school thus lays the groundwork for advanced topics in mathematics.

Overemphasized and Underemphasized Topics

There is remarkable consistency among the states in topics that are overemphasized and underemphasized.

In general, we found too little attention paid to the coherent development of fractions in the late elementary and early middle school grades, and not enough emphasis on paper-and-pencil calculations. A related topic at the high school level that deserves much more emphasis is the arithmetic of rational functions. This is crucial for students planning university studies in math-related majors, including engineering and the physical and biological sciences. They will need facility in addition, subtraction, multiplication, and division of rational functions, including long division of polynomials. The most important prerequisite for this frequently missing topic in state standards is the arithmetic of fractions. Many state standards would also benefit from greater emphasis on completing the square of quadratic polynomials, including a derivation of the quadratic formula, and applications to graphs of conic sections.

Among topics that receive too much emphasis in state standards are patterns, use of manipulatives, estimation, and probability and statistics. We discuss each of these in turn.

Patterns

The attention given to patterns in state standards verges on the obsessive. In a typical state document, students are asked, through a broad span of grade levels, to create, identify, examine, describe, extend, and find "the rule" for repeating, growing, and shrinking patterns, as well as where the patterns may be found in numbers, shapes, tables, and graphs. Thus, first-graders in Maryland are required to "recognize the difference between patterns and non-patterns." How this is to be done, and what

Fig. 4: State Grades in Descending Order

STATE	Clarity	Content	Reason	Negative Qualities	Final G.P.A.	2005 GRADE
CA	3.83	3.94	3.83	3.92	3.89	A
IN	3.67	3.83	4.00	3.75	3.82	A
MA	3.67	3.67	2.00	3.50	3.30	A
AL	3.00	3.17	2.00	3.50	2.97	B
NM	3.00	2.67	2.00	3.00	2.67	B
GA	3.33	2.67	2.00	2.00	2.53	B
WV	2.00	2.50	3.00	1.75	2.35	C
NY	1.50	2.33	2.00	2.25	2.08	C
MI	2.17	1.67	2.00	2.50	2.00	C
AZ	2.00	2.00	2.00	2.00	2.00	C
OK	2.17	1.83	1.50	2.50	1.97	C
VA	2.83	2.00	1.50	1.50	1.97	C
NC	2.33	1.50	1.50	2.25	1.82	C
SD	2.17	1.67	1.00	2.50	1.80	C
TX	2.67	1.67	1.00	2.00	1.80	C
IL	1.50	2.00	1.00	2.50	1.80	C
KY	1.83	2.33	1.00	1.50	1.80	C
ND	2.33	1.33	1.00	3.00	1.80	C
LA	2.00	2.33	1.00	1.25	1.78	C
MD	2.00	1.67	1.50	2.00	1.77	C
NV	2.17	1.33	1.50	2.50	1.77	C
TN	1.83	1.33	2.00	2.00	1.70	D
MN	2.00	1.67	1.00	2.00	1.67	D
MS	1.33	2.00	1.00	2.00	1.67	D
Average	1.85	1.57	1.15	1.79	1.59	D
WI	1.67	1.67	1.00	1.50	1.50	D
OH	2.00	1.33	1.00	1.50	1.43	D
NE	1.72	1.28	0.67	2.17	1.42	D
CO	1.00	1.67	1.00	1.50	1.37	D
DC	1.67	1.33	1.50	1.00	1.37	D
ME	1.17	1.17	0.50	2.75	1.35	D
OR	2.50	1.00	0.00	2.25	1.35	D
AK	2.00	1.17	0.50	1.75	1.32	D
SC	1.00	1.67	1.50	0.75	1.32	D
PA	1.33	1.17	1.00	1.75	1.28	D
VT	1.33	1.00	0.67	2.00	1.20	D
NJ	2.17	1.17	0.50	0.75	1.15	D
UT	1.83	1.17	0.50	1.00	1.13	D
ID	1.67	0.67	1.00	1.50	1.10	D
MT	1.00	1.00	0.00	2.00	1.00	D
WY	1.00	0.83	0.00	2.25	0.98	F
FL	1.33	0.67	1.50	0.50	0.93	F
KS	1.67	0.94	0.33	0.25	0.83	F
AR	1.50	0.67	0.00	0.75	0.72	F
NH	1.17	0.67	0.00	1.00	0.70	F
RI	1.00	0.67	0.00	1.00	0.67	F
MO	0.67	0.33	1.00	0.50	0.57	F
WA	0.33	1.00	0.50	0.00	0.57	F
DE	0.83	0.67	0.50	0.00	0.54	F
CT	0.67	0.33	0.00	1.00	0.47	F
HI	1.00	0.33	0.00	0.50	0.43	F
IA	-	-	-	-	-	-

(A = 4.00 - 3.25; B = 3.24 - 2.50; C = 2.49 - 1.75; D = 1.74 - 1.00; F = 0.99 - 0.00) exactly is meant by a pattern, is anyone's guess. Florida's extensive requirements for the study of patterns call upon second-graders to use "a calculator to explore and solve number patterns"; identify "patterns in the real-world (for example, repeating, rotational, tessellating, and patchwork)"; and explain "generalizations of patterns and relationships," among other requirements.

The following South Dakota fourth-grade standard is an example of false doctrine (a notion explained in greater detail on page 34) that is representative of standards in many other state documents.

> *Students are able to solve problems involving pattern identification and completion of patterns. Example: What are the next two numbers in the sequence? Sequence: ...*

The sequence "1, 3, 7, 13, __, __" is then given. The presumption here is that there is a unique correct answer for the next two terms of the sequence, and by implication, for other number sequences, such as: 2, 4, 6, __, ___, and so forth. How should the blanks be filled for this example? The pattern might be continued in this way: 2, 4, 6, 8, 10, etc. But it might also be continued this way: 2, 4, 6, 2, 4, 6, 2, 4, 6. Other continuations include: 2, 4, 6, 4, 2, 4, 6, 4, 2, or 2, 4, 6, 5, 2, 4, 6, 5. Similarly, for the example in the South Dakota standard, the continuation might proceed as 1, 3, 7, 13, 21, 31, or as 1, 3, 7, 13, 1, 3, 7, 13, or in any other way. Given only the first four terms of a pattern, there are infinitely many systematic, and even polynomial, ways to continue the pattern, and there are *no possible incorrect* fifth and sixth terms. Advocating otherwise is both false and confusing to students. Such problems, especially when posed on examinations, misdirect students to conclude that mathematics is about mind reading: To get the correct answer, it is necessary to know what the teacher wants. Without a rule for a pattern, there is no mathematically correct or incorrect way to fill in the missing numbers.

Typical strands in state standards documents are "Patterns, Functions, and Algebra," "Patterns and Relationships," "Patterns, Relations, and Algebra," "Patterns and Relationships," and so forth. As these strand titles suggest, there is a tendency among the states to conflate the study of algebra with the exploration of patterns. For example, Wyoming's entire "Algebraic Concepts and Relationships" strand for fourth grade consists of three standards, all devoted to the study of patterns:

> 1. *Students recognize, describe, extend, create, and generalize patterns by using manipulatives, numbers, and graphic representations.*
> 2. *Students apply knowledge of appropriate grade level patterns when solving problems.*
> 3. *Students explain a rule given a pattern or sequence.*

An obscure Montana high school algebra standard requires students to "use algebra to represent patterns of change." South Carolina's seventh-graders are asked to:

> *Explain the use of a variable as a quantity that can change its value, as a quantity on which other values depend, and as generalization of patterns.*

The convoluted standard above illustrates several generic deficiencies of state algebra standards. The notion that algebra is the study of patterns is not only wrong, it shrouds the study of algebra in mystery and can lead to nonsensical claims like the one here, that a variable is "a generalization of patterns." Beginning algebra should be understood as generalized arithmetic. A letter such as "x" is used to represent only a number and nothing more. Computation with an expression in x is then the same as ordinary calculations with specific, familiar numbers. In this way, beginning algebra becomes a natural extension of arithmetic, as it should.

We are not arguing that standards calling upon students to recognize patterns should be eliminated. For example, it is desirable that children recognize patterns associated with even or odd numbers, be able to continue arithmetic and geometric sequences, and be able to express the nth terms of such sequences and others algebraically. Recognizing patterns can also aid in problem-solving or in posing conjectures. Our point here is that the attention given to patterns is excessive, sometimes destructive, and far out of balance with the actual importance of patterns in K-12 mathematics.

Manipulatives

Manipulatives are physical objects intended to serve as teaching aids. They can be helpful in introducing new concepts for elementary students, but too much use runs the risk that the students will focus on the manipulatives more than the mathematics, and even come to depend on them. Ultimately, the goal of elementary school math is to get students to manipulate numbers, not objects, in order to solve problems.

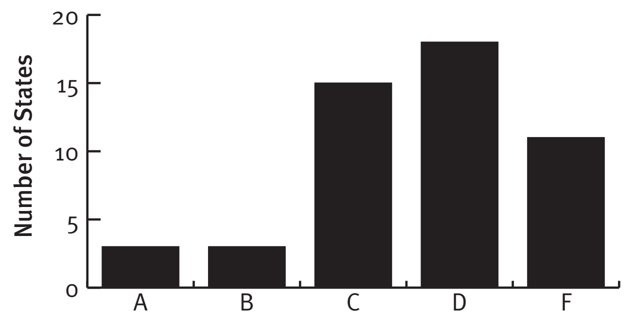
In higher grades, manipulatives can undermine important educational goals. There may be circumstances when a demonstration with a physical object is appropriate, but ultimately *paper and pencil are by far the most useful and important manipulatives.* They are the tools that students will use to do calculations for the rest of their lives. Mathematics by its very nature is abstract, and it is abstraction that gives mathematics its power.

Yet many state standards documents recommend and even require the use of a dizzying array of manipulatives for instruction or assessment in counterproductive ways. New Jersey's assessment requires that students be familiar with a collection of manipulatives that includes base ten blocks, cards, coins, geoboards, graph paper, multi-link cubes, number cubes (more commonly known as dice), pattern blocks, pentominoes, rulers, spinners, and tangrams. Kansas incorrectly refers to manipulatives as "Mathematical Models," and uses that phrase 572 times in its framework. The vast array of physical devices that Kansas math students must master includes place value mats, hundred charts, base ten blocks, unifix cubes, fraction strips, pattern blocks, geoboards, dot paper, tangrams, and attribute blocks. It is unclear in these cases whether students learn about manipulatives in order to better understand mathematics, or the other way around.

New Jersey and Kansas are far from unique in this regard. According to Alabama's introduction to its sixth-grade standards, "The sixth-grade curriculum is designed to maximize student learning through the use of manipulatives, social interaction, and technology." In New Hampshire, eighth-graders are required to "perform polynomial operations with manipulatives." Eighth-graders in Arkansas must "use manipulatives and computer technology (e.g., algebra tiles, two color counters, graphing calculators, balance scale model, etc.) to develop the concepts of equations."

The requirement to use algebra tiles in high school algebra courses is both widespread and misguided. Rather than requiring the use of plastic tiles to multiply and factor polynomials, states should insist that students become adept at using the distributive property, which is vastly more powerful and much simpler.

Figure 5: Final Grade Distribution, 2005



Estimation

Fostering estimation skills in students is a commendable goal shared by all state standards documents. However, there is a tendency to overemphasize estimation at the expense of exact arithmetic calculations. Idaho provides a useful illustration. Its first- and second-grade standards prematurely introduce estimation and "reasonableness" of results. These skills are more appropriately developed in the higher grades, after students have experience with exact calculations. In the elaboration of one first-grade standard, this example is provided: "Given *9 - 4*, would 10 be a reasonable number?" Similarly, for second grade, one finds: "Given subtraction problem, *38 - 6*, would 44 be a reasonable answer?" These examples are misguided. For these subtractions, the correct answer is the only reasonable answer. The notion of "reasonableness" might be addressed in grades 1 and 2 in connection with measurement, but not in connection with arithmetic of small whole numbers. Care should be taken not to substitute estimation for exact calculations.

Probability and Statistics

With few exceptions, state standards documents at all grade levels include strands of standards devoted to probability and statistics. Standards of this type almost invariably begin in Kindergarten (and sometimes pre-Kindergarten). Utah, for example, asks its Kindergartners to "understand basic concepts of probability," an impossible demand since probabilities are numbers between 0 and 1 and Kindergartners do not have a clear grasp of fractions. Perhaps in recognition of this, Utah's Kindergarten requirement includes the directive, "Relate past events to future events (e.g., The sun set about 6:00 last night, so it will set about the same time tonight)." But how such a realization about sunsets contributes to understanding basic concepts of probability is anyone's guess. Probability standards at the Kindergarten level are unavoidably ridiculous. In a similar vein, Vermont's first-graders are confronted with this standard:

For a probability event in which the sample space may or may not contain equally likely outcomes, use experimental probability to describe the likelihood or chance of an event (using "more likely," "less likely").

Again, this is premature and pointless. There is nothing to be gained by introducing the subject of probability to students who do not have the prerequisites to understand it. The state report cards that follow are full of similar examples.

Coherent mathematics standards delay the introduction of probability until middle school, and then proceed quickly by building on students' knowledge of fractions and ratios. Indiana does not have a probability and statistics strand for grades K-3. Other states would do well to emulate that commendable feature and carry it further by postponing most of their elementary school probability standards until middle school.

Many states also include data collection standards that are excessive. New York's third- and fourth-graders, for example, are required to:

Make predictions, using unbiased random samples.

- *Collect statistical data from newspapers, magazines, polls.*
- *Use spinners, drawing colored blocks from a bag, etc.*
- *Explore informally the conditions that must be checked in order to achieve an unbiased random sample (i.e., a set in which every member has an equal chance of being chosen) in data gathering and its practical use in television ratings, opinion polls, and marketing surveys.*

The time used for such open-ended activities would be better spent on mathematics.

Statistics and probability requirements typically appear with standards for all other mathematical topics, and often crowd out important topics in algebra and geometry. For example, West Virginia's Algebra I students are required to "perform a linear regression and use the results to predict specific values of a variable, and identify the equation for the line of regression," and to "use process (flow) charts and histograms, scatter diagrams, and normal distribution curves." Conflating geometry with statistics, Texas sixth-graders are required to "generate formulas to represent relationships involving perimeter, area, volume of a rectangular prism, etc., from a table of data." Statistical explorations should not replace a coherent geometric development of perimeter, area, and volume. Mississippi's Algebra II students "use scatter plots and apply regression analysis to data." While not always identified in the short state reports that follow, standards requiring visual estimation of lines or curves of best fit for statistical data are abundant in middle and high school algebra and geometry courses. Finding the coefficients for lines of best fit is college-level mathematics and is best explained at that level. The K-12 alternatives are to ask students to "eye ball" lines of best fit, or merely press calculator buttons without understanding what the machines are doing. Students would be better off learning, for example, rational function arithmetic, or how to complete the square for a quadratic polynomial—topics frequently missing or abridged.

Mathematical Reasoning and Problem-Solving

Problem solving is an indispensable part of learning mathematics and, ideally, straightforward practice problems should gradually give way to more difficult problems as students master skills. Unfortunately, few states offer standards that guide the development of problem-solving in a useful way. Students should solve single-step word problems in the earliest grades and deal with increasingly more challenging, multi-step problems as they progress.

As important as problem-solving is, there is much more to mathematical reasoning than solving word problems alone. Fordham I presents an illuminating discussion of mathematical reasoning in K-12 mathematics that includes this elaboration:

> *The beauty and efficacy of mathematics both derive from a common factor that distinguishes mathematics from the mere accretion of information, or application of practical skills and feats of memory. This distinguishing feature of mathematics might be called mathematical reasoning, reasoning that makes use of the structural organization by which the parts of mathematics are connected to each other, and not just to the real world objects of our experience, as when we employ mathematics to calculate some practical result.*[2]

The majority of states fail to incorporate mathematical reasoning directly into their content standards. Even for high school geometry, where it is difficult to avoid mathematical proofs, many state documents do not ask students to know proofs of anything in particular. Few states expect students to see a proof of the Pythagorean Theorem or any other theorem or any collection of theorems. Mathematical proofs should also be integrated into algebra and trigonometry courses, but it is a rare state that asks students even to know how to derive the quadratic formula in a high school algebra course.

Mathematical reasoning should be an integral part of the content at all grade levels. For example, elementary and middle school geometry standards should ask students to understand how to derive formulas for areas of simple figures. Students should be guided through a logical, coherent progression of formulas by relating areas of triangles to areas of rectangles, parallelograms, and trapezoids. But many states expect only that children will compute areas when given correct formulas. An example—one of many—is this North Dakota seventh-grade standard:

> *Students, when given the formulas, are able to find circumference, perimeter, and area of circles, parallelograms, triangles, and trapezoids (whole number measurements).*

Not only does this standard not ask for understanding of the basic area formulas, students aren't even asked to achieve the modest goal of memorizing them. We note also that the restriction in this standard to whole numbers is unnecessary and counterproductive at the seventh grade level, when knowledge of the arithmetic of real numbers, including pi, is clearly assumed in this very instruction.

Fig. 6: Changes in State Grades, 2000 - 2005

Big Improvement	Small Improvement	Same	Small Decline	Big Decline
IN	ID	AL	AZ	DE
LA	IL	AK	AR	DC
MA	MN	CA	CT	KS
MI	ND	CO	FL	MS
NM	TN	GA	KY	NH
		HI	NE	NC
		ME	NJ	OH
		MD	NY	SC
		MO	OK	SD
		MT	PA	UT
		NV	TX	
		OR	VT	
		RI	VA	
		WA	WV	
			WI	
			WY	

NOTE: Big improvement (or decline) signifies movement of more than one letter grade.

The logical development of fractions and decimals deserves special attention, rarely given in state documents. In many cases, students are inappropriately expected to multiply and divide decimal numbers a year in advance of multiplying and dividing fractions. This is problematic. What does it *mean* to multiply or divide

[2] *State Math Standards*, by Ralph Raimi and Lawrence Braden, Thomas B. Fordham Foundation, March 1998, page 9.

decimal numbers, if those operations for fractions have not been introduced? How are these operations defined? All too often, we found no indication that students should understand multiplication and division of rational numbers except as procedures.

In many cases, reliance on technology replaces mathematical reasoning. An example is this Ohio standard for seventh grade:

> *Describe differences between rational and irrational numbers; e.g., use technology to show that some numbers (rational) can be expressed as terminating or repeating decimals and others (irrational) as non-terminating and non-repeating decimals.*

The technology is not specified, but calculators cannot establish the fact that rational numbers necessarily have repeating or terminating decimals. On the other hand, the characterization of decimal expansions of rational numbers can be made in a straightforward manner using the long division algorithm.

Mathematical reasoning is systematically undermined when prerequisites for content standards are insufficiently developed. When arithmetic, particularly fraction arithmetic, is poorly developed in the elementary grades, students have little hope of understanding algebra as anything other than a maze of complicated recipes to be memorized, as is too often the case in state standards documents.

Perhaps the most strident denial of the importance of prerequisites in mathematics appears in Hawaii's Framework:

> *Learning higher-level mathematics concepts and processes are [sic] not necessarily dependent upon "prerequisite" knowledge and skills. The traditional notion that students cannot learn concepts from Algebra and above (higher-level course content) if they don't have the basic skill operations of addition, subtraction, etc. has been contradicted by evidence to the contrary.*

Unsurprisingly, no such evidence is cited for this wrong headed assertion. Prerequisites cannot be discarded. They are essential to mathematics. The failure to develop appropriate prerequisites and mathematical reasoning based on those prerequisites leads to the degeneration of mathematics standards into what might be described as mathematics appreciation. Hawaii is part of an unfortunate trend among the states to introduce calculus concepts too early and without necessary prerequisites. Thus, Hawaiian fourth graders are asked to identify and describe "situations with varying rates of change such as time and distance [*sic*]." Likewise, with no development of calculus prerequisites, one of Maryland's algebra standards is:

> *The student will describe the graph of a non-linear function and discuss its appearance in terms of the basic concepts of maxima and minima, zeros (roots), rate of change, domain and range, and continuity.*

Pennsylvania's Framework even has a strand entitled "Concepts of Calculus," which lists standards for each of the grades 3, 5, 8, and 11. Fifth-graders are supposed to "identify maximum and minimum." This directive is given without specifying the type of quantity for which extrema are to be found, or any method to carry out such a task. Pennsylvania's eleventh-grade standards under this strand also have little substance. Without any mention of limits, derivatives, or integrals, and no further elaboration, they require students to "determine maximum and minimum values of a function over a specified interval" and "graph and interpret rates of growth/decay."

Similarly out of place and unsupported by any discussion of derivatives is the South Carolina Algebra II standard: "Determine changes in slope relative to the changes in the independent variable." But perhaps the most bizarre of what might be termed "illusory calculus" standards is this New Mexico grade 9-12 standard:

> *Work with composition of functions (e. g., find f of g when* f(x) = 2x - 3 *and* g(x) = 3x - 2)*, and find the domain, range, intercepts, zeros, and local maxima or minima of the final function.*

We note that there is no hint of calculus in any of the New Mexico grade 9-12 standards except for this one. Further, why restrict the identification of local extreme values only to compositions of functions? Compounding the confusion, since these two functions *f(x)* and *g(x)* are linear, their composition is also linear, and there are no maximum or minimum values of that composition.

The failure to fully recognize prerequisites as essential to learning mathematics not only leads to premature coverage of calculus topics, but opens the floodgates for superficial content standards. For example, a Missouri standard (under the heading of "What All Students Should Be Able To Do") absurdly asks high school students to,

> *Evaluate the logic and aesthetics of mathematics as they relate to the universe.*

Similar examples of inflation appear in many state standards.[3]

The Roots of, and Remedy for, Bad Standards

Why are so many state standards documents of such low quality? What factors influence their content? What accounts for the uniformity of their flaws?

The National Council of Teachers of Mathematics (NCTM) has had, and continues to have, immense influence on state education departments and K-12 mathematics education in general. Many state standards adhere closely to guidelines published by the NCTM in a long sequence of documents. Three have been especially influential: *An Agenda for Action* (1980), *Curriculum and Evaluation Standards for School Mathematics* (1989), and *Principles and Standards for School Mathematics* (2000). We refer to the latter two documents respectively as the 1989 NCTM Standards and the 2000 NCTM Standards.

An Agenda for Action was the blueprint for the later documents, paving the way for current trends when it called for "decreased emphasis on such activities as . . . performing paper-and-pencil calculations with numbers of more than two digits." This would be possible, the document explained, because "the use of calculators has radically reduced the demand for some paper-and-pencil techniques." Accordingly, "all students should have access to calculators and increasingly to computers throughout their school mathematics program." This includes calculators "for use in elementary and secondary school classrooms." Regarding basic skills, the report warned, "It is dangerous to assume that skills from one era will suffice for another." *An Agenda for Action* further stressed that "difficulty with paper-and-pencil computation should not interfere with the learning of problem-solving strategies." Foreshadowing another trend among state standards documents, the 1980 report also encouraged "the use of manipulatives, where suited, to illustrate or develop a concept or skill."

Fig. 7: Changes in State Grades, 1998 - 2005

Big Improvement	Small Improvement	Same	Small Decline	Big Decline
IN	ID	AL	AK	DE
LA	IL	AR	AZ	MS
MD	ME	CA	CT	NH
MA	NE	CO	FL	NC
MI	ND	DC	KY	OH
NM		GA	NJ	UT
OK		HI	NY	
SD		MO	TN	
		OR	TX	
		PA	VT	
		RI	VA	
		SC	WV	
		WA	WI	

NOTE: Big improvement (or decline) signifies movement of more than one letter grade.

The 1989 NCTM Standards amplified and expanded *An Agenda for Action*. It called for some topics to receive increased attention in schools and other topics to receive decreased attention. Among the grade K-4 topics slated for greater attention were "mental computation," "use of calculators for complex computation," "collection and organization of data," "pattern recognition and description," and "use of manipulative materi-

[3] "Inflation" is one of two subcategories of the "negative qualities" criterion used in the evaluation of standards documents. See the section, *Criteria for Evaluation,* page 31.

als." The list of topics recommended for decreased attention included "complex paper-and-pencil computations," "long division," "paper and pencil fraction computation," "rote practice," "rote memorization of rules," and "teaching by telling." For grades 5-8, the 1989 NCTM Standards took an even more radical position, recommending for de-emphasis "manipulating symbols," "memorizing rules and algorithms," "practicing tedious paper-and-pencil computations," and "finding exact forms of answers."

Like *An Agenda for Action*, the 1989 NCTM Standards put heavy emphasis on calculator use at all grade levels. On page 8, it proclaimed, "The new technology not only has made calculations and graphing easier, it has changed the very nature of mathematics" and recommended that "appropriate calculators should be available to all students at all times."

The influence of the 1989 NCTM Standards on state standards can hardly be overstated. After the publication of Fordham I, author Ralph Raimi wrote:

> *These state standards, though federally encouraged and supported, are supposed to be each state's vision of the future, of what mathematics education ought to be. Some were apparently written by enormous committees of teachers and math education specialists, but the final texts obviously were assembled and organized at the state education department level sometimes with the help of one of the regional educational "laboratories" set up and financed by the U.S. Department of Education. Despite the regional differences, the influence of NCTM and these laboratories has imparted a certain sameness to many of the state standards we ended up studying. Almost all of them had publication dates of 1996 or 1997.*[4]

Many of the documents evaluated in this Fordham report were also published, or drafted, prior to the appearance of the 2000 NCTM Standards.

The 1989 NCTM Standards document was the subject of harsh criticism during the 1990s. As a consequence, some of the more radical declarations of the 1989 document were eliminated in the revised 2000 NCTM Standards. However, the latter document promoted the same themes of its predecessors, including emphasis on calculators, patterns, manipulatives, estimation, nonstandard algorithms, etc. Much of the sameness of current state standards documents may be traced to the NCTM's vision of mathematics education.

A fuller explanation for the shortcomings of state math standards, however, goes beyond the influence of the NCTM and takes into account the deficient mathematical knowledge of many state standards authors. Mathematical ignorance among standards writers is the greatest impediment to improvement.

Some guidelines for improving standards, based on this report, suggest themselves immediately. States can correct the "common problems" identified in this essay, such as overuse of calculators and manipulatives, overemphasis of patterns and probability and statistics, and insufficient development of the standard algorithms of arithmetic and fraction arithmetic. But here the devil is in the details and these corrections should not be attempted by the people who created the problems in the first place. For the purpose of writing standards, there is no substitute for a thorough understanding of mathematics—not mathematics education or pedagogy, but the subject matter itself. A state education department's usual choice of experts for this task will likely cause as many new problems as it solves.

Of particular importance is a coherent and thorough development of arithmetic in the early grades, both in terms of conceptual understanding *and* computational fluency. Without a solid foundation in this most important branch of mathematics—arithmetic—success in secondary school algebra, geometry, trigonometry, and pre-calculus is impossible. The challenges in developing credible arithmetic standards should not be underestimated. Standards authors lacking a deep understanding of mathematics, including advanced topics, are not up to the task.

A simple and effective way to improve standards is to adopt those of one of the top scoring states: California, Indiana, or Massachusetts. At the time of this writing,

[4] "Judging State Standards for K-12," by Ralph Raimi, Chapter 2 in *What's at Stake in the K-12 Standards Wars: A Primer for Educational Policy Makers*, edited by Sandra Stotsky, Peter Lang Publishing, page 40.

the District of Columbia was considering replacing its standards with the high quality standards from one of these states. That makes good sense. There is no need to reinvent the wheel. The goal of standards should not be innovation for its own sake; the goal is to implement useful, high-quality standards, regardless of where they originated.

Four Antidotes to Faulty State Standards

1. **Replace the authors of low-quality standards documents with people who thoroughly understand the subject of mathematics. Include university professors from mathematics departments.**

2. **Develop coherent arithmetic standards that emphasize both conceptual understanding *and* computational fluency.**

3. **Avoid the "common problems" described above, such as overuse of calculators and manipulatives, overemphasis of patterns and probability and statistics, and insufficient development of the standard algorithms of arithmetic and fraction arithmetic.**

4. **Consider adopting a complete set of high-quality math standards from one of the top scoring states: California, Indiana, or Massachusetts.**

If, however, a state chooses to develop its own standards in whole or in part, some university level mathematicians (as distinguished from education faculty) should be appointed to standards writing committees and be given enough authority over the process so that their judgments cannot easily be overturned. Such a process was used in California in December 1997 and resulted in the highest-ranked standards in all three Fordham math standards evaluations. The participation of university math professors in the development of K-12 standards is becoming increasingly important. Since 1990, more than 60 percent of high school graduates have gone directly to colleges and universities[5] and that percentage is likely to increase. College preparation should therefore be the default choice (though not the only option) for K-12 mathematics. For this purpose, the perspective of university mathematics professors on what is needed in K-12 mathematics to succeed in college is indispensible.

[5] National Center for Education Statistics, Table 183 – College enrollment rates of high school completers, by race/ethnicity: 1960 to 2001.