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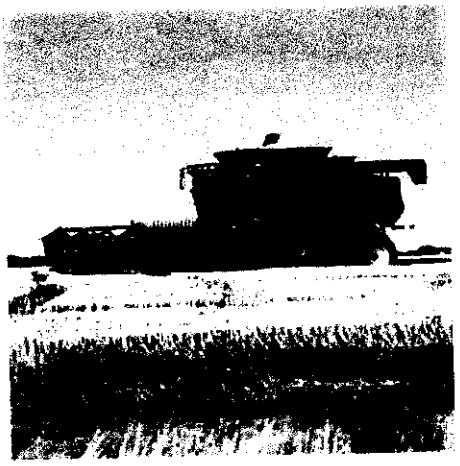
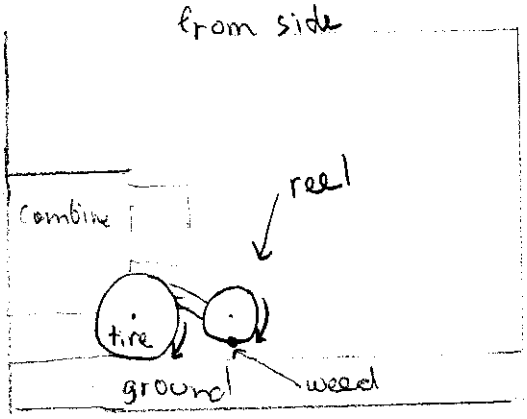
Math 221, Cardwell 129

"Combine(d) Motion"

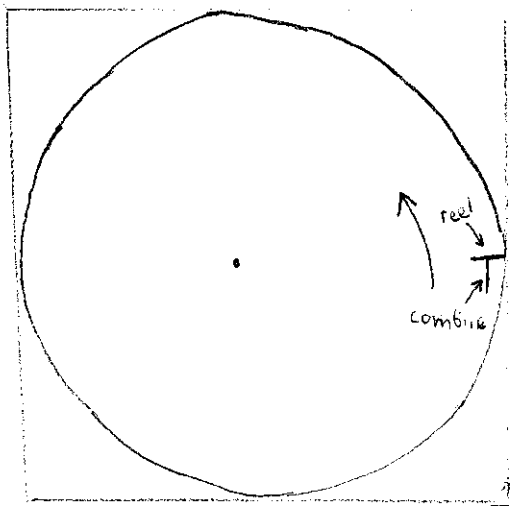
A combine is cutting wheat in a field. The field is an irrigated circle that is one mile across. The combine is going 6 m.p.h. The reel on the combine, spinning clockwise, spins around 4 times for every 3 times the combine goes around. The radius of the reel is 15 inches and the radius of the combine tire is 2.5 feet. The bottom of the reel is 9 inches above the ground. If the field was on a map, north being up, the combine will start cutting the most eastern point of the field and cut counterclockwise. The reel on the combine is 30 feet wide. At time 0, a weed gets stuck at the bottom of the reel on the right side, 15 feet to the right of the combine. Find the parametric equation for the position of the weed.



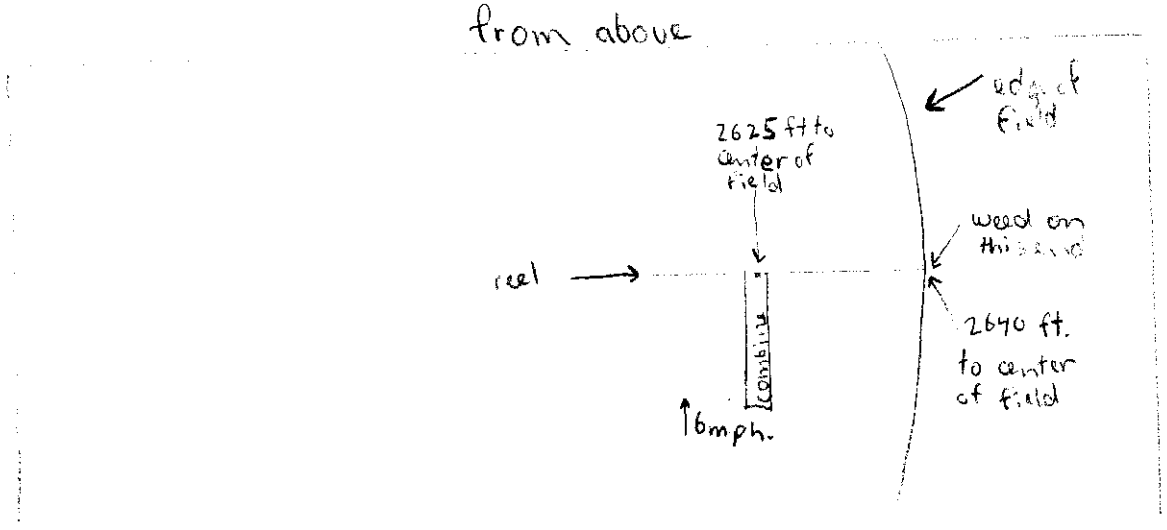
from side



from above



from above



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❶ - combine goes $\frac{6\text{m}}{\text{h}} \left| \frac{5280\text{ft}}{1\text{m}} \right| \frac{1\text{hour}}{3600\text{s}} = 8.8\ \text{ft/sec}$

- bottom of reel is 9" above ground, reel has radius 15" ∴ z_c is 2' above ground

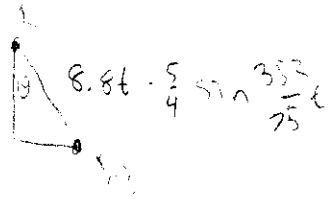
$$\begin{pmatrix} V_x \\ V_z \end{pmatrix} = \begin{pmatrix} 8.8 \\ 0 \end{pmatrix} \qquad x_{cr} = 8.8t + C \qquad y_{cr} = 0t + C \qquad x_0 = 0 \qquad z_0 = 2 \qquad \therefore \begin{pmatrix} x(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} 8.8t \\ 2 \end{pmatrix}$$

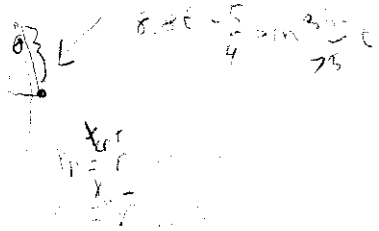
- large tire on front of combine has radius 2.5 ft

$$x_c(t) = r\,\theta(t)$$

$$\frac{8.8t}{2.5} = \theta(t)$$

$$\frac{8.8}{2.5} = \frac{d\theta_t}{dt}$$



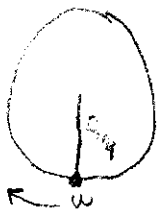


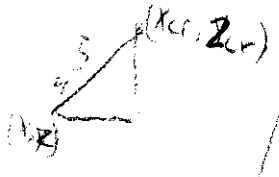
- reel makes 4 revolutions for every 3 tire revolutions

$$3x = 4 \qquad x = \frac{4}{3} \qquad \frac{d\theta_r}{dt} = \frac{8.8}{2.5}\left(\frac{4}{3}\right)$$

$$\theta(t) = \int \frac{352}{75} = \frac{352t}{75} + C$$

- weed gets stuck at bottom of reel at $t=0$





8.85 comes from next page ↓

$$x = x_{cr} - r\sin\theta = 8.85t - \frac{5}{4}\sin\left(\frac{352t}{75}\right)$$

$$z = z_{cr} - r\cos\theta = 2 - \frac{5}{4}\cos\left(\frac{352t}{75}\right)$$

↑ height

→ = distance weed is from center of reel when looking from above

- combine cuts on circle with radius ½ mile (2640 ft), going counter clockwise
  - center of combine (going 8.8 ft/sec) is on radius 2625 ft, ↳ starting at (2625, 0)
  - end of reel is 15 feet to right of center of combine [2640, 0]

- speed of the end of the reel

V_1 = combine
V_2 = end of reel

$$\frac{V_1}{r_1} = \frac{V_2}{r_2} \qquad V_2 = \frac{V_1 r_2}{r_1} = \frac{8.8 \text{ ft/sec} \cdot (2640 \text{ ft})}{2625 \text{ ft}} = 8.85 \text{ ft/sec}$$

looking ↓ from above

Velocity at center of reel $= 8.85t + \frac{\pi}{2}$

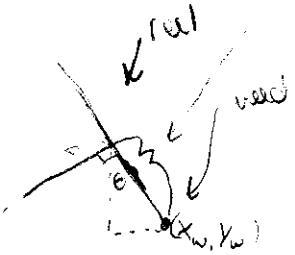
$x_r\ y_r$

2640

- θ

$x_c = 2640 \cos\theta$
$y_c = 2640 \sin\theta$

$8.8t - \frac{5}{4}\sin\left(\frac{352t}{75}\right)$



8.85 ft/sec

2640 ft | reel | 8.85 ft ?

$\int \frac{5280 \text{ ft}}{8.85} \text{ sec} = 2\pi \text{ rad}$

$\frac{5280}{17.7} = \theta$

$$x_w = x_{cr} + r\sin\theta$$
$$y_w = x_{cr} - r\cos\theta$$

$$x_w = 2640\cos\theta + \left(8.85t - \frac{5}{4}\sin\left(\frac{352t}{75}\right)\right)\sin t$$
$$y_w = 2640\sin\theta - \left(8.85t - \frac{5}{4}\sin\left(\frac{352t}{75}\right)\right)\cos\theta$$

$$\begin{cases} x_w = 2640\cos\left(\frac{5280}{17.7}t\right) + \left(8.85t - \frac{5}{4}\sin\left(\frac{352}{75}t\right)\right)\sin\left(\frac{5280}{17.7}t\right) \\ y_w = 2640\sin\left(\frac{5280}{17.7}t\right) + \left(8.85t - \frac{5}{4}\sin\left(\frac{352t}{75}\right)\right)\cos\left(\frac{5280t}{17.7}\right) \\ z_w = 2 - \frac{5}{4}\cos\left(\frac{352t}{75}\right) \end{cases}$$