Homework 9 Due Friday @ 6pm

1) Find the volume of the 3-dimensional region which is under the plane $z = 1 + 2y$ and above the bounded region of the xy-plane which is enclosed by $y = x^2$ and $y = 4$. $\boxed{\text{Ans } \frac{316}{5}}$

2) Use a double integral in polar coordinates to calculate the volume of the 3-dimensional region enclosed by the surfaces $z = x^2 + y^2$ and $z = 6 - \sqrt{x^2 + y^2}$. $\boxed{\text{Ans } \frac{32\pi}{3}}$

3) Find the volume of the region which is under the surface $z = x + xy$ and above the region in the first quadrant of the xy-plane that is bounded by $y = 2x$, $y = 0$, and $x = 2$. $\boxed{\text{Ans } \frac{40}{3}}$

4) Evaluate the integral $\iint_R xy\, dA$ where R is the region in the first quadrant enclosed by the parabolas $4y = x^2$ and $4x = y^2$. $\boxed{\text{Ans } \frac{64}{3}}$

5) Find the volume of the 3-D region enclosed by the surfaces $y = x^2$, $y = 4$, $z = 5 + x$, $z = 2x$. $\boxed{\text{Ans } \frac{160}{3}}$

6) A mass distribution occupies the region which is above $z = x^2 + y^2$ and under $z = 4$. If the mass density function is $\delta(x, y, z) = z^2$ units of mass/unit volume, calculate the total mass. $\boxed{\text{Ans } 64\pi}$

7) Find the volume of the 3-dimensional region enclosed by the surfaces $y = x^2$, $y = 4$, and $z = 10 - y$.
$\boxed{\text{Ans } \frac{1216}{15}}$

8) Calculate the area of that part of the surface $z = y^2$ which is above the region of the xy-plane bounded by $y = x$, $x = 0$, and $y = 1$.
$\boxed{\text{Ans } \frac{1}{12}(5\sqrt{5} - 1)}$

9) Define the the tetrahedron

$$T = \{(x, y, z) \in \mathbb{R}^3_+ | x + y + z \leq 1\}.$$

Notice that this just forms an object with vertices (0,0,0), (1,0,0), (0,1,0), and (0,0,1). Find the volume of T and find the center of mass (the centroid) of T. $\boxed{\text{Ans Volume} = \frac{1}{6}, \text{ centroid } (\frac{1}{4}, \frac{1}{4}, \frac{1}{4})}$

10) A mass distribution is in the region above the cone $\phi = \pi/4$ and inside the sphere $\rho = 2$. The mass density function is $\delta = z$. Use a triple integral in spherical coordinates to find the total mass. $\boxed{\text{Ans } 2\pi}$

11) Find the area of that part of the plane $3x - 4y + 5z = 10$ which is between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 100$. $\boxed{\text{Ans } 99\pi\sqrt{2}}$

12) A mass distribution occupies the region in the first octant enclosed by the surfaces $z = x$, $z = 0$, $y = 1$, $y = x^2$ and $x = 1$. If the mass density function is $\delta = \dfrac{1}{1 + x^2}$ units of mass/unit of volume, calculate the total mass. $\boxed{\text{Ans } -\frac{1}{2} + \ln 2}$

13) Let R be a region in the plane and $f(x,y)$ a function of 2-variables. What is the area of R if

$$\iint_R f(x,y)dA = 26$$

and

$$\iint_R (f(x,y)+2)dA = 108$$

Ans 41

14) Find the area of that part of the surface $z = 1 + x^2$ which lies above the region of the xy-plane which is enclosed by the curves $y = x$, $y = 0$ and $x = 2$. Ans $\frac{1}{12}[17\sqrt{17} - 1]$

15) A mass distribution occupies the 3 dimensional region which is above the cone $z = \sqrt{x^2 + y^2}$ and below the plane $z = 4$. If the mass density function is $\delta = z$ units of mass/unit volume, use a triple integral in **spherical coordinates** to calculate the total mass. Ans 64π

16) A piece of surface Σ is described in cylindrical coordinates as $z = r\sin\theta$ where $0 \le r \le 1$ and $0 \le \theta \le \pi$. Find the area of Σ. [Remember, you can look up $|\vec{N}|$ in cylindrical, but here $dS = |\vec{N}|drd\theta$, and **not** $|\vec{N}|rdrd\theta$, that's the point of this problem.] Ans $\frac{\pi}{\sqrt{2}}$

17) A mass distribution occupies the region which is above the surface $z = \sqrt{x^2 + y^2}$ and under the plane $z = 2$. The mass density function is $\delta = z\sqrt{x^2 + y^2 + z^2}$ units of mass/unit volume. Use a triple integral in **spherical coordinates** to calculate the total mass. Ans $\dfrac{64\pi}{15}(2\sqrt{2} - 1)$

18) Find the volume under $z = e^{-(x^2+y^2)}$ by first calculating volume over a disk of radius R and then let R go to infinity. Ans π

19) Use spherical coordinates to integrate

$$\iiint_Q \frac{1}{x^2 + y^2 + z^2 + (x^2 + y^2 + z^2)^2}\, dV$$

where Q is the ball of radius 1. Ans π^2

20) Find the centroid of the solid bounded by the surfaces $z = y$, $y = x^2$, $y = 4$, and $z = 0$. Ans $(0, \frac{20}{7}, \frac{10}{7})$

21) $\displaystyle\int_0^{1/16}\int_{y^{1/4}}^{1/2} \cos(16\pi x^5)\, dxdy$ Ans $\frac{1}{80\pi}$

22) $\displaystyle\int_0^8\int_{\sqrt[3]{x}}^2 \frac{dydx}{y^4 + 1}$ Ans $\frac{1}{4}\ln 17$