

Homework 4. Due Friday Feb 29 @ 6pm

Pictures:

Draw pictures of the following surfaces in 3-space.

- 1.1) $z = x^2 + y^2$
- 1.2) $z = \sqrt{x^2 + y^2}$
- 1.3) $z = -x$
- 1.4) $x = z^2$
- 1.5) $y = 1$
- 1.6) $z = x^2 - y^2$
- 1.7) $x^2 + y^2 = 4$

Chain rule problems:

2.1) A quantity Q depends on x and y according to $Q = xe^{x^2y}$. Both x and y are changing with time t . Suppose at a certain instant you know that $x = 2$, $y = -1$, $dx/dt = 3$ and $dy/dt = 1/2$. Use the chain rule to find dQ/dt at this instant.

2.2) Given that $z = f(x, y)$ with x and y functions of s and t by $x = st$, $y = s^3 - t^3$ with $s > 0$ and $t > 0$. Suppose you know that at $(x, y) = (4, 0)$, $\frac{\partial f}{\partial x}(4, 0) = 2$ and $\frac{\partial f}{\partial y}(4, 0) = 1$. Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ when $(x, y) = (4, 0)$.

Find and classify critical points:

- 3.1) Find and classify the critical points for $f(x, y) = x^3 + 3xy^2 - 6xy$.
- 3.2) Find and classify the critical points of $f(x, y) = 4xy - y^4 - x^2$.
- 3.3) Find and classify the critical points of $f(x, y) = x^3y^2 + 2xy - 2x$.
- 3.4) Find and classify the critical points of $f(x, y) = \frac{1}{2}x^2y - \frac{1}{2}x^2 + \frac{1}{3}y^3 - \frac{3}{2}y^2$.
- 3.5) Let $f(x, y) = 12xy - x^2y - y^3$. Find and classify the critical points of $f(x, y)$.
- 3.6) Find and classify the critical points of $f(x, y) = 4xy - xy^2 - x^3$.