

Recommended Extra Problems to study for Exam 3.

1) A mass distribution occupies the region which is between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$. If the mass density function is $\delta(x, y, z) = z^4$ units of mass/unit volume, find the total mass. Ans $\frac{508\pi}{35}$

2) Find the volume of the 3-dimensional region enclosed by the surfaces $y = x^2$, $y = 4$, $z = y$ and $z = 10 - y$. Ans $\frac{832}{15}$

3) Use a triple integral in spherical coordinates to calculate the total mass of a mass distribution which lies above the cone $\phi = \cos^{-1}\left(\frac{4}{5}\right)$ and under the plane $z = 2$, given that the mass density function is $\delta = z^2$ units of mass/unit volume. Ans $\frac{18\pi}{5}$

4) A mass distribution occupies the region in the 1st octant enclosed by the surfaces $z = x$, $z = 0$, $y = 1$, $y = 1 + x^2$ and $x = 2$. If the mass density function is $\delta = \frac{4}{1 + x^2}$ units of mass/unit volume, calculate the total mass. Ans $8 - 2 \ln 5$

5) Find the area of that part of the plane $2x + y + 4z = 8$ which is in the 1st octant and is between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. Ans $\frac{3}{16}\pi\sqrt{21}$

6) A mass distribution occupies the region in the first octant which is enclosed by the surfaces $y = x$, $x = 1$, $y = 0$, $z = 1 + x$, $z = 0$. If the mass density function is $\delta = x^2 + y^2$ units of mass/unit volume, calculate the total mass. Ans $\frac{9}{15}$

7) Find the volume of the 3-dimensional region which is inside the sphere $x^2 + y^2 + z^2 = 2$ and above the paraboloid $z = x^2 + y^2$. Ans $\frac{\pi}{6}(8\sqrt{2} - 7)$

8) A mass distribution occupies the region in 3-space enclosed by the surfaces $z = \sqrt{x^2 + y^2}$ and $z = 2$. Its mass density function is $\delta(x, y, z) = x^2 + y^2 + z^2$. Use a triple integral in **spherical coordinates** to calculate the total mass. Ans $\frac{48\pi}{5}$

9) Find the volume of the 3 dimensional region under $z = 1 + x$ and above the region of the xy -plane which is bounded by the curves $x = 0$, $y = 4$ and $y = x^2$. Ans $\frac{28}{3}$

10) Find the volume of the 3 dimensional region which is enclosed by the surfaces $y = x^2$, $y = x$, $z = 6$ and $z = y$. Ans $\frac{14}{15}$

11) Evaluate $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{1+y^3} dy dx$ by first **reversing the order of integration**. Ans $\frac{2}{3} \ln 3$

12) A mass distribution occupies the 3 dimensional region which is above $z = x^2 + y^2$ and below the plane $z = 4$. If the mass density function is $\delta = \sqrt{x^2 + y^2}$ units of mass/unit volume, calculate the total mass. Ans $\frac{128\pi}{15}$

13) A mass distribution occupies the 3-d region which is above the xy -plane and between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 16$. If the mass density function is $\delta(x, y, z) = (x^2 + y^2)z$ units of mass/unit volume, calculate the total mass. Ans 336π

14) Find the center of mass of a solid of constant density δ bounded by $z = 0$ and $z = 4 - x^2 - y^2$. Ans $(0, 0, \frac{4}{3})$

15) Find the centroid of the region that is bounded above by the surface $z = \sqrt{r}$, on the sides by the cylinder $r = 4$, and below by the xy -plane. Ans $(0, 0, \frac{5}{6})$

16) Find the z -coordinate of the center of mass of the upper hemisphere of radius 2002, given that the density function of this solid is $\delta = z^{2002}$. Ans $\bar{z} = \frac{2002 \cdot 2003 \cdot 2005}{2004 \cdot 2006}$