Answers to Exam 2 Practice Problems

Chain rule problems:

1)$dQ/dt = -17e^{-4}$
2) $z_s = 16$, $z_t = -8$

Gradients and directional derivatives:

1) a) $\nabla f = \left(6\sqrt{x+y^2} + \dfrac{3x}{\sqrt{x+y^2}}, \dfrac{6xy}{\sqrt{x+y^2}}\right)$
b) $-\dfrac{26}{\sqrt{5}}$
c) $\frac{1}{\sqrt{929}}(23, 20)$
2) $\dfrac{-5}{\sqrt{21}}$
3) a) $\nabla f = (y^2 e^{xy}, e^{xy} + xye^{xy})$
b) $-\frac{2}{e^2\sqrt{37}}$
c) $(4/\sqrt{17}, -1/\sqrt{17})$

Tangent planes given by the gradient: a) $5(x+1) + 11(y-2) - 10(z-1) = 0$
b) $x + 4y - 3z = 0$
c) $-5(x-1) + 13(y-2) - 2(z-3) = 0$
d) $33(x-1) + 37(y-2) + 35(z-3) = 0$
e) $29(x-3) + 15(y-2) + 18(z-1) = 0$
f) $23(x-5) + 20(y-2) - 3(z-30) = 0$

Lagrange multipliers:

1) $f(-\sqrt{18}, -3) = 9\sqrt{2} \leftarrow$ max, $f(-\sqrt{18}, 3) = -9\sqrt{2} \leftarrow$ min, $f(\sqrt{18}, -3) = -9\sqrt{2} \leftarrow$ min, $f(\sqrt{18}, 3) = 9\sqrt{2} \leftarrow$ max.
2) $f(4,0) = 24 \leftarrow$ max, $f(-4,0) = 8 \leftarrow$ nothing, $f(-1/2, \pm\sqrt{63/4}) = -66/4 \leftarrow$ min.
3) $f(2,4,0) = 10 \leftarrow$ nothing, $f(-2,-4,0) = -10 \leftarrow$ min, $f(1,2,\pm\sqrt{15/2}) = 12.5 \leftarrow$ max.
4) $f(4,0) = 28/3 \leftarrow$ nothing, $f(-4,0) = -28/3 \leftarrow$ min, $f(3, \pm\sqrt{7}) = 7 \leftarrow$ nothing, $f(-1, \pm\sqrt{15}) = 53/3 \leftarrow$ max.

Find and classify critical points:

1) (0,0) saddle; (0,2) saddle; (1,1) local min; $(-1,1)$ local max.
2) (0,0) saddle; $(2\sqrt{2}, \sqrt{2})$ local max; $(-2\sqrt{2}, -\sqrt{2})$ local max.
3) (0,1) saddle; $(1/\sqrt{2}, -2)$ saddle; $(-1/\sqrt{2}, -2)$ saddle.
4) (0,0) local max; (0,3) local min; (2,1) saddle; $(-2,1)$ saddle.
5) (0,0) saddle; (12,0) saddle; $(6, \sqrt{12})$ local max; $(6, -\sqrt{12})$ local min.
6) (0,0) saddle; (0,4) saddle; $(2/\sqrt{3}, 2)$ local max; $(-2/\sqrt{3}, 2)$ local min.

Volumes and areas using double integrals:

1) $\int_0^2 \int_{2x}^4 (1+xy)\,dydx = 12$

2) $\int_0^2 \int_{x^2}^4 [(8-y)-3]\,dydx = \frac{208}{15}$

3) $\int_0^{2\pi} \int_0^2 (8-2r^2)r\,drd\theta = 16\pi$

4) $\int_0^2 \int_0^{2x} (x+xy)\,dydx = \frac{40}{3}$

5) $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} xy\,dydx = \frac{128}{6}$

6) $\int_{-2}^2 \int_{x^2}^4 (1+2y)\,dydx = \frac{928}{15}$

7) $\int_0^{2\pi} \int_0^2 (6-r-r^2)r\,drd\theta = \frac{32\pi}{3}$

Reversing the order of integration:

1) $\int_0^2 \int_{y^2}^4 \sqrt{1+x\sqrt{x}}\,dxdy = \int_0^4 \int_0^{\sqrt{x}} \sqrt{1+x\sqrt{x}}\,dydx = \frac{104}{9}$

2) $\int_0^4 \int_{\sqrt{x}}^2 \sqrt{1+y^3}\,dydx = \int_0^2 \int_0^{y^2} \sqrt{1+y^3}\,dxdy = \frac{52}{9}$

3) $\int_0^2 \int_{x^2}^4 \frac{2x}{1+y^2}\,dydx = \int_0^4 \int_0^{\sqrt{y}} \frac{2x}{1+y^2}\,dxdy = \frac{1}{2}\ln 17$