Answers to the Fall 2000 Final Exam.

$\boxed{1}$ (a) $(2,-1,1)$

(b) $\sqrt{11}$

(c) $\frac{1}{\sqrt{11}}(1,1,3)$

(d) $-\frac{3}{\sqrt{11}}$

(e) $\sqrt{\frac{90}{11}}$

(f) $\frac{1}{11}\sqrt{\frac{90}{11}}$

(g) it is slowing down because $a_T < 0$

$\boxed{2}$ (a) $\vec{v}(t) = (1, t, \sin t), \quad \vec{r}(t) = (t, \frac{t^2}{2}, 2 - \cos t)$

(b) $x = t$, $y = \frac{t^2}{2}$, $z = 2 - \cos t$

(c) $\dfrac{\sqrt{2+4\pi^2}}{(1+4\pi^2)^{3/2}}$

$\boxed{3}$ $-\dfrac{2e^2}{5}$

$\boxed{4}$
$f(0,2) = 0$
$f(0,-2) = 8 \longleftarrow \text{max}$
$f(\pm\frac{1}{2}\sqrt{15}, \frac{1}{2}) = -\frac{18}{4} \longleftarrow \text{min}$

$\boxed{5}$ (a) $\frac{3}{2}$

(b) $\theta = 135°$

(c) $(2,1,2)\cdot(x-1,y,z-1)=0$

$\boxed{6}$ $\frac{508}{35}\pi$

$\boxed{7}$ $\frac{832}{15}$

$\boxed{8}$ $\frac{1}{12}(5^{3/2}-1)$

$\boxed{9}$ $\frac{1}{3\sqrt{2}}(17^{3/2}-1)$

$\boxed{10}$ $\frac{1048}{15}$

$\boxed{11}$ Solution: $\text{div}\vec{F}=2+e^z$

$$\iiint_T (\text{div}\vec{F})\,dV = \iiint_T (2+e^z)dV = \iint_R \int_0^{4-r^2} (2+e^z)dzdA$$

$$= ... = \int_0^{2\pi}\int_0^2 (8-2r^2+e^{4-r^2}-1)rdrd\theta = ... = 11\pi+\pi e^4.$$

Let S_1 be the top parabaloid and S_2 be the bottom plane. Then,

$$\iint_{S_1} \vec{F}\cdot\vec{n}\,dS = \iint_R (x(2x)+y(2y)+e^z)dxdy = \int_0^{2\pi}\int_0^2 (2r^2+e^{4-r^2})rdrd\theta$$

$$= ... = 15\pi+\pi e^4.$$

$$\iint_{S_2} \vec{F}\cdot\vec{n}\,dS = \iint_{S_2}(-e^z)dS = \iint_{S_2}(-1)dS = -\text{area}(S_2) = -4\pi.$$

Thus the total outward flux is $15\pi+\pi e^4+(-4\pi)=11\pi+\pi e^4$.

Note that on the above two surface integrals you can plug in what z is on the surface. For example, on S_1, $z=4-x^2-y^2$; and on S_2, $z=0$.