Answers to the Fall 1999 Exam 1.

1 (a) $\frac{1}{2}\sqrt{35}$

(b) $\theta = \cos^{-1}(-\frac{1}{6}) \approx 99.6^\circ$

(c) $5(x-1) - y + 3(z-2) = 0$

(d) $\vec{r}(t) = (5, -1, 3)t + (1, 0, 2)$

2 (a) False. Since $a_T = 0$, we have $\vec{a} = a_N \vec{N}$. And $a_N = \kappa v^2 = 9\kappa$, note $\kappa \neq 0$ for an ellipse. Thus $|\vec{a}| = 9\kappa \neq 0$.

(b) False. $|\vec{v}| = 3 \neq 0$.

(c) True. $|\vec{v}| = 3 \iff \vec{v} \cdot \vec{v} = 9 \Longrightarrow \vec{a} \cdot \vec{v} + \vec{v} \cdot \vec{a} = 0 \iff \vec{a} \cdot \vec{v} = 0$

(d) False. At each instant the acceleration vector is perpendicular to the velocity vector. So they can't be parallel.

3 $\vec{v}(t) = (1, 2t+1, -1+\cos t), \quad \vec{r}(t) = (t+1, t^2+t, -t+\sin t)$

$x = t + 1,$
$y = t^2 + t,$
$z = -t + \sin t$

4 (a) $\vec{r} = (\sin t, \cos t, \sin 2t)$

(b) $\vec{v} = (\cos t, -\sin t, 2\cos 2t)$

(c) $\vec{a} = (-\sin t, -\cos t, -4\sin 2t)$

(d) $|\vec{v}| = \sqrt{1 + 4\cos^2 2t}$

(e) $a_T = D_t|\vec{v}| = \dfrac{\vec{a}\cdot\vec{v}}{|\vec{v}|} = \dfrac{-8\cos 2t \sin 2t}{\sqrt{1+4\cos^2 2t}}$

(f) $\kappa = \dfrac{|\vec{a}\times\vec{v}|}{|\vec{v}|^3} = \dfrac{1}{5}$ at $t = 0$.

(e) $a_N = \kappa|\vec{v}|^2 = 1$ at $t = 0$.

$\boxed{5}$ (a) $f_x = 6xy - 12x^2$
$f_{xx} = 6y - 24x$
$f_y = 3x^2 - 6y^2 + 7$
$f_{yy} = -12y$
$f_{xy} = f_{yx} = 6x$

(b) $-6(x-1) + 4(y-1) - (z-5) = 0$

(c) $(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}})$ $\quad (\frac{-1}{\sqrt{3}}, \frac{-2}{\sqrt{3}})$