

CALCULUS III

NAME _____

EXAM III

Rec. Instr. _____

SPRING 1997

Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

- (10) 1. Find the equation of the plane which is tangent to the surface $x^3y + y^3z + z^3x = 5$ at the point $(x, y, z) = (1, -1, 2)$.

(20) 2. Let $f(x, y) = x^2e^{xy}$.

Find

a) the gradient vector field of f .

b) the directional derivative of $f(x, y)$ at $(x, y) = (1, 2)$ in the direction of the vector $\vec{a} = -\vec{i} + \vec{j}$.

c) a unit vector which points in the direction which gives the largest directional derivative of f at $(1, 2)$.

d) the value of the largest directional derivative of f at $(1, 2)$.

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(20) 3. Find and classify the critical points for the function

$$f(x, y) = y(x^2 + 2x - 3) - \frac{y^2}{2}$$

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(10) 4. Use the method of Lagrange multipliers to find the maximum value and the minimum value of $f(x, y) = xy$ on the ellipse $x^2 + 3y^2 = 12$.

- (15) 5. Find the volume of the 3-dimensional region which is under the surface $z = 3 + x$ and above the region of the xy -plane which is enclosed by the curves $y = x^2$ and $y = 4$.

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(10) 6. Evaluate the iterated integral below *by first reversing the order of integration*.

$$\int_0^4 \int_{\sqrt{x}}^2 9 \cos(y^3) dy dx$$

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- (15) 7. Find the volume of the 3-dimensional region which is bounded by the paraboloids $z = 6 - x^2 - y^2$ and $z = x^2 + y^2$. Use polar coordinates to evaluate the double integral.