CALCULUS III NAME ______________________

EXAM 3 Rec. Instr. ______________________

SPRING 1996 Rec. Time ______________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(10) 1. A quantity Q depends upon x and y according to $Q = x\sqrt{x+y^2}$. Suppose that x and y are changing with time t and at $t = 2$ seconds you know that

$$x = 5, \quad y = 2, \quad \frac{dx}{dt} = -3 \quad \text{and} \quad \frac{dy}{dt} = 3.$$

Use the chain rule to find $\frac{dQ}{dt}$ at $t = 2$.

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(15) 2. Suppose that $z = f(x,y)$ and at the point $(x,y) = (1,2)$ you know that $\frac{\partial z}{\partial x}(1,2) = 3$, $\frac{\partial z}{\partial y}(1,2) = 4$. If r and θ denote polar coordinates in the xy-plane, **use the chain rule** to find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ at $(x,y) = (1,2)$.

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(15) 3. Let $f(x,y,z) = x^2y + y^2 + xz^3$.

Find

(a) The gradient vector field of f .

$\bigtriangledown f =$

(b) The directional derivative of f at the point $(x,y,z) = (-1,1,2)$ in the direction of $\vec{a} = 2\vec{i} - \vec{j} + 2\vec{k}$.

(c) The value of the largest directional derivative of f at $(-1,1,2)$. In which direction does it occur?

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(15) 4. A wire in the form of the ellipse $x^2 + 2y^2 = 8$ is heated in such a way that its temperature at (x, y) is given by $T = x^2 + 4y^2 + 2y + 3$. Use the method of **Lagrange Multipliers** to find the hottest and coldest points on the wire and the corresponding hottest and coldest temperatures.

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(20) 5. Find the critical points for $f(x, y) = -x^2y + 2xy - y^3$. Use the 2nd derivative test on each critical point to see if it gives a local max, a local min, or a saddle point.

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(10) 6. Evaluate $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{1+y^3} dydx$ by first reversing the order of integration.

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(15) 7. Use a double integral to calculate the volume of the bounded region in the first octant which is enclosed by the surfaces

$$z = 1 + 2xy, \quad y = x^2, \quad y = 2x, \quad \text{and} \quad z = 0.$$