CALCULUS III NAME _____

EXAM III Rec. Instr. _____

FALL 1998 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. A mass distribution occupies the region in the first octant which is enclosed by the surfaces $y = x^2$, $y = x$, $z = 1$, $z = 2 + x$. If the mass density function is $\delta(x, y, z) = 10y$ units of mass/unit volume, calculate the total mass in the region.

NAME _____

Rec. Instr. _____

(15) 2. A mass distribution occupies the region which is above the surface $z = \sqrt{x^2 + y^2}$ and under the plane $z = 2$. If the mass density function is $\delta(x, y, z) = z^2$, use a triple integral in spherical coordinates to calculate the total mass.

NAME _____

Rec. Instr. _____

(15) 3. Calculate the area of that part of the paraboloid $z = x^2 + y^2$ which is between the planes $z = 1$ and $z = 9$.

NAME _____

Rec. Instr. _____

(15) 4. The force field $\vec{F} = y\vec{i} - x\vec{j}$ acts on an object as it moves in the plane. Calculate the work done by $\vec{F}$ as the object moves from (0,0) to (3,1) by going from (0,0) to (1,1) along the curve $y = x^2$ and then along the line segment from (1,1) to (3,1).

NAME _____

Rec. Instr. _____

(10) 5. Show that the force field

$$\vec{F} = \left(y - \frac{1}{x^2}\right)\vec{i} + \left(x - \frac{1}{y^2}\right)\vec{j}$$

is conservative in the region $x > 0$ and $y > 0$ by finding a potential function for $\vec{F}$. Now use the potential function to find the work done by $\vec{F}$ as it acts on an object which moves along a curve from (1,1) to (3,2) in the region $x > 0$ and $y > 0$.

NAME _____

Rec. Instr. _____

(15) 6. Use Green's theorem to evaluate the line integral

$$\int_C (1 + xy)\, dx + (x^2 + y) dy$$

where C is the triangle with vertices (0,0), (2,0), (0,2) directed counter-clockwise.

NAME _____

Rec. Instr. _____

(15) 7. Evaluate the surface integral $\int\int_S \sqrt{x^2+y^2}\, dS$ where S is the parametrized surface $x = s\cos t\,,\ y = s\sin t\,,\ z = t$ for $0 \le s \le 1\,,\ 0 \le t \le 2\pi\,.$