CALCULUS III — NAME ______________________

EXAM 3 — Rec. Instr. ______________________

FALL 2001 — Rec. Time ______________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK

(15) 1. A mass distribution occupies the region in the 1st octant which is bounded by the planes $y = 0$, $x = 2$, $y = x$, $z = 0$, and $z = y$. The mass density function is $\delta(x, y, z) = 15z^2$ units of mass/unit volume. Calculate the total mass.

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(15) 2. A mass distribution occupies the region which is inside the sphere $x^2 + y^2 + z^2 = 4$ and above the plane $z = 0$. The mass density function is $\delta(x, y, z) = x^2 + y^2 + z^2$ units of mass/unit volume. Use a triple integral <u>in spherical coordinates</u> to calculate the total mass.

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(25) 3. The force field $\vec{F} = (x - y)\vec{\imath} + x\vec{\jmath}$ acts on an object as it moves in the plane. Calculate the work done by $\vec{F}$ for each of the following motions.

a) The object moves from $(0, 0)$ to $(1, 1)$ along the curve $x = y^2$.

b) The object moves from $(0, 0)$ to $(1, 1)$ by going first along the x-axis from $(0, 0)$ to $(1, 0)$ and then along the vertical line segment from $(1, 0)$ to $(1, 1)$.

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(10) 4. Let $\vec{F} = xy^2\vec{i} + y\vec{j}$. Use the vector form of Green's theorem to calculate the outward flux, $\int_C \vec{F} \cdot \vec{n}\, ds$, where C is the circle $x^2 + y^2 = 4$.

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(10) 5. Show that the force field

$$\vec{F} = \left(2x + \frac{1}{y}\right)\vec{i} + \left(2 - \frac{x}{y^2}\right)\vec{j}$$

is conservative in the region $y > 0$ by finding a potential function for it. Now use this potential function to calculate the work done by $\vec{F}$ as it gets on an object which moves from $(0, 3)$ to $(4, 2)$ along any curve which lies in the upper half plane.

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(15) 6. Let S be the parametrized surface $x = t$, $y = t^2$, $z = st$, $0 \le s \le 4$, $1 \le t \le 3$.

a) Calculate the area of S.

b) Evaluate the surface integral $\displaystyle\int\int_S \left(\frac{z}{x}\right)\, dS$.

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(15) 7. Evaluate the surface integral $\displaystyle\int\int_S \vec{F} \cdot \vec{n}\, dS$ where S is that part of $z = 5 - x^2 - y^2$ which is above the plane $z = 1$, $\vec{n}$ is the upward pointing unit normal vector, and $\vec{F}$ is the vector field $\vec{F} = x\vec{i} + z\vec{k}$.