

ANALYTIC GEOMETRY & CALCULUS III

Exam 2

July 6, 1995

NAME _____ Rec. Instr. _____

Signature _____ Class Time _____

To get full credit your answers should be accurate and precise.

1. Suppose that $z = f(x, y)$ and that $x = u^3 + v^3$, $y = \frac{u}{v}$. Suppose that at $(x, y) = (16, 1)$ we know that $\frac{\partial f}{\partial x}(16, 1) = 1$ and that $\frac{\partial f}{\partial y}(16, 1) = 6$. Use the Chain Rule to calculate the values of $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ at $(x, y) = (16, 1)$.

2. Describe the graph of $\rho = 2 \sin \phi$.

3. Sketch some level curves of the function

$$f(x, y) = \exp(-x^2 - y^2).$$

4. Let

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0). \end{cases}$$

Show that $f(x, y)$ is not continuous at $(0, 0)$.

5. Write an equation of the plane tangent to

$$z = \sin \frac{\pi xy}{2}, \quad \text{at the point } p = (3, 5, -1).$$

6. Find the dimensions x , y , and z of a rectangular box with fixed volume $V = 1000$ and minimum total surface area A .

7. A rectangular box with no top is to have a bottom made from material that costs $\$3/\text{ft}^2$ and sides made from material that costs $\$1/\text{ft}^2$. The box is to have total volume 12 ft^3 . Find the minimum possible cost.

8. Let $f(x, y, z) = \frac{4x^2y}{z}$.

(a) Find the directional derivative of $f(x, y, z)$ at the point $(1, 3, 2)$ in the direction of the vector $i - 2j + 2k$.

(b) Find the value of the largest directional derivative of $f(x, y, z)$ at $(1, 3, 2)$.

(c) Find a unit vector which points in the direction that gives the largest directional derivative of $f(x, y, z)$ at $(1, 3, 2)$.

9. Suppose that $w = f(u) + g(v)$ when $u = x - at$, and that $v = x + at$. Show that

$$\frac{\partial^2 w}{\partial t^2} = a^2 \frac{\partial^2 w}{\partial x^2}.$$

10. Use differentials to approximate $\Delta f = f(Q) - f(P)$, where

$$f(x, y) = \sqrt{x^2 + y^2}; \quad P(3, 4), \quad Q(2.97, 4.04).$$