CALCULUS III NAME _____

EXAM II Rec. Instr. _____

SPRING 1998 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. An object is moving in 3-space according to the parametric equations $x = t^2$, $y = \sin t$, $z = \cos t$. Find a_T, a_N and the curvature κ as functions of the time t.

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(15) 2. An object is moving in the plane along the curve $y = \frac{1}{2}x^2$. It is moving from left to right at a constant speed of 4 ft/sec.

a) Find a_T and a_N when the object is at the point $\left(x, \frac{1}{2}x^2\right)$.

b) Find the velocity vector and the acceleration vector when the object is at the point $\left(1, \frac{1}{2}\right)$.

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(15) 3. An object is moving in the plane. At a certain instant, say $t = 2$ seconds, you know that $\vec{r}(2) = \vec{i} - 2\vec{j}$, $\vec{v}(2) = 2\vec{i} + \vec{j}$ and $\vec{a}(2) = -\vec{i} + \vec{j}$ for the position vector, velocity vector and acceleration vector respectively. Answer the following questions. Do not attempt to find $\vec{r}$, $\vec{v}$ and $\vec{a}$ as functions of time.

At $t = 2$ seconds,

a) where is the object located?

b) what is the speed of the object?

c) what is the unit tangent vector $\vec{T}$?

d) what is a_T?

e) what is a_N?

f) Is the object speeding up or slowing down? Why?

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(20) 4. Let $f(x, y) = 3x^2y + 2x^3 + 3y^2 + 1$.

a) Find the equation of the plane tangent to $z = f(x, y)$ at the point $(1, -2, 9)$.

b) Find all points on the surface $z = f(x, y)$ at which the tangent plane is horizontal.

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(15) 5. A quantity Q depends upon x and y according to $Q = xe^{x^2y}$. Both x and y are changing with the time t and at a certain instant you know that $x = 2$, $y = \dfrac{1}{2}$, $\dfrac{dx}{dt} = 2$ and $\dfrac{dy}{dt} = -2$. Use the chain rule to find $\dfrac{dQ}{dt}$ at this instant.

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(15) 6. Suppose that $z = f(x, y)$ and at the point $(x, y) = (2, 3)$ you know that $\dfrac{\partial f}{\partial x}(2, 3) = 4$ and $\dfrac{\partial f}{\partial y}(2, 3) = 2$. If (r, θ) denote polar coordinates in the xy plane, use the chain rule to calculate $\dfrac{\partial z}{\partial \theta}$ and $\dfrac{\partial z}{\partial r}$ when $x = 2$, $y = 3$.

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(5) 7. Determine whether $f(x,y) = \frac{1}{2}\ln(x^2+y^2)$ satisfies the partial differential equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$, by actually calculating the partial derivatives involved.