CALCULUS III NAME ______________________

EXAM II Rec. Instr. ______________________

FALL 1997 Rec. Time ______________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. A quantity Q depends upon x and y according to $Q = xe^{x^2 y}$. Both x and y are changing with time t. Suppose at a certain instant you know that $x = 2$, $y = -1$, $\frac{dx}{dt} = 3$ and $\frac{dy}{dt} = \frac{1}{2}$. Use the chain rule to find $\frac{dQ}{dt}$ at this instant.

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(15) 2. Given that $z = f(x,y)$ with x and y functions of s and t by $x = st$, $y = s^3 - t^3$ with $s > 0$ and $t > 0$. Suppose you know that at $(x,y) = (4,0)$, $\frac{\partial f}{\partial x}(4,0) = 2$ and $\frac{\partial f}{\partial y}(4,0) = 1$. Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ when $(x,y) = (4,0)$.

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(20) 3. Let $f(x,y) = 6x\sqrt{x+y^2}$.

Find

a) the gradient vector field of f, $\nabla f =$

b) the directional derivative of f at $(x,y) = (5,2)$ in the direction of the vector $\vec{a} = -2\vec{\imath} + \vec{\jmath}$.

c) the value of the largest directional derivative of f at $(x,y) = (5,2)$ and a unit vector which points in the direction which gives this largest directional derivative.

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(15) 4. Find the equation of the tangent plane to the given surface at the specified point.

a) $xyz^2 + x^3 + y^3 - 2z^3 = 3$ at $(-1, 2, 1)$

b) $z = \sqrt{x^2 + 2y^2}$ at $(1, 2, 3)$

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(15) 5. Use the method of Lagrange multipliers to find the maximum value and the minimum value of $f(x,y) = xy$ on the ellipse $x^2 + 2y^2 = 36$.

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(20) 6. Find and classify the critical points for $f(x, y) = x^3 + 3xy^2 - 6xy$.