

CALCULUS III

NAME _____

EXAM 2

Rec. Instr. _____

FALL 1996

Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. Given that $z = f(x, y)$ and that x and y are functions of s and t according to $x = s^2 - t^2$, $y = st$, $t > 0$. Suppose you know that, at the point $(x, y) = (0, 4)$, $\frac{\partial f}{\partial x}(0, 4) = 3$ and $\frac{\partial f}{\partial y}(0, 4) = 4$. Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ when $x = 0$, $y = 4$.

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(20) 2. Let $f(x, y) = y\sqrt{x^2 + y^2}$. Find

a) the gradient vector field of f

$$\nabla f =$$

b) the directional derivative of f at $(x, y) = (3, 4)$ in the direction of the vector $\vec{a} = \vec{i} - 2\vec{j}$

c) the value of the largest directional derivative of f at $(3, 4)$

d) a unit vector which points in the direction which gives the largest directional derivative of f at $(3, 4)$

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(15) 3. Find the equation for the tangent plane to the given surface at the specified point.

a) the graph of $f(x, y) = x^3y - xy^2 + y^2$ at $(1, 2, 2)$

b) $xy^3 + x^2z + yz^3 = 41$ at $(2, 1, 3)$

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(20) 4. Find and classify the critical points for $f(x, y) = 8y^3 + 12x^2 - 24xy$.

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- (15) 5. Use the method of Lagrange multipliers to find the maximum value and the minimum value of $f(x, y) = 3x - 2y$ on the ellipse $x^2 + 2y^2 = 44$.

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(15) 6. Determine (by calculating the partial derivatives and checking directly) whether the given functions $u(x, t)$ satisfy the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

a) $u(x, t) = e^{-4t} \sin x \cos x$

b) $u(x, t) = e^{-3t} \sin 2x$