

Name \_\_\_\_\_

Rec. Instr. \_\_\_\_\_

CALCULUS III

Exam 2

Fall 1995

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

- (15) 1. A quantity  $Q$  depends upon  $x$  and  $y$  according to  $Q = x^2 e^{xy}$ . Suppose  $x$  and  $y$  are changing with time  $t$  and at  $t = 2$  seconds we know that  $x = 2$ ,  $y = 1$ ,  $\frac{dx}{dt} = -3$  and  $\frac{dy}{dt} = 4$ . Use the **Chain Rule** to find  $\frac{dQ}{dt}$  at  $t = 2$  seconds.

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- (15) 2. Suppose that the temperature at the point  $(x, y, z)$  with distance measured in kilometers is given by

$$T(x, y, z) = 5 + xy + xz + yz$$

in degrees Celsius. Find the rate of change (in degrees per kilometer) of temperature at the point  $P(2, 1, 3)$  in the direction of the vector  $\mathbf{v} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ .

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(10) 3. Suppose the temperature  $T$  (in degrees Celsius) at the point  $(x, y)$  is given by

$$T(x, y) = 10 + \frac{1}{2}x^2 + \frac{1}{4}y^2.$$

In what direction  $\mathbf{u}$  (a unit vector) should a bumblebee at the point  $(3, 4)$  initially fly in order to get warmer fastest? Find the directional derivative  $D_{\mathbf{u}}f(3, 4)$  in this optimal direction  $\mathbf{u}$ .

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(15) 4. Find the equations of the following planes.

(a) Tangent plane to the graph of  $f(x, y) = \frac{5x}{x+y}$  at the point  $(2, 3, 2)$ .

(b) Tangent plane to the surface

$$xyz + x^2 - 2y^2 + z^3 = 14$$

at the point  $(5, -2, 3)$ .

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- (20) 5. Find the critical points for the function  $f(x, y) = \frac{4}{3}x^3 + 4y^3 - x^4 - y^4$ .  
Find the highest point on the graph of  $f(x, y)$ . (This graph has a highest point but no lowest point.)

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(15) 6. Use the method of Lagrange multipliers to find the maximum value and the minimum value of  $f(x, y) = xy$  on the ellipse  $4x^2 + y^2 = 16$ .

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(10) 7. Show that  $u(x, y) = \sqrt{x^2 + y^2}$  satisfies the equation

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 1$$