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(20) 1. Let $F(x, y, z) = x\sqrt{x^2 + y^2 + z^2}$

a) Find the gradient vector field of F ,

$$\nabla F =$$

b) Find the directional derivative of $F(x, y, z)$ at $(x, y, z) = (3, 4, 12)$ in the direction of the vector $\vec{v} = -\vec{i} + \vec{j} + \vec{k}$.

c) Find a unit vector which points in the direction that $F(x, y, z)$ is changing most rapidly at the point $(0, 3, 4)$. What is the value of the directional derivative of $F(x, y, z)$ in this direction at $(0, 3, 4)$?

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(20) 2. Use the **method of Lagrange multipliers** to find the largest value and the smallest value of $f(x, y) = \frac{1}{3}x^3 - 2x - y^2$ on the ellipse $x^2 + 2y^2 = 9$.

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(25) 3. Find and classify the critical points of $f(x, y) = 12xy - x^2y - y^3$.

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- (15) 4. Find the volume of the 3D-region which is under $z = x^2 + 2y$ and above the region in the 1st quadrant of the xy -plane that is bounded by the lines $y = 2x$, $y = 4$ and $x = 0$.

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(10) 5. Evaluate $\int_0^2 \int_{x^2}^4 (1 + y\sqrt{y})^{\frac{1}{2}} dy dx$ by **first reversing the order of integration**.

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- (10) 6. Calculate the volume of the 3D-region which is enclosed by the surfaces $z = 12 - x^2 - y^2$ and $z = 8$. Use polar coordinates to evaluate the double integral.