

# CALCULUS III

## Exam 1

June 21, 1995

NAME \_\_\_\_\_ Rec. Instr. \_\_\_\_\_

Signature \_\_\_\_\_ Class Time \_\_\_\_\_

To get full credit your answers should be accurate and precise.

1. Compute  $a_T$  and  $a_N$  at the point  $\left(1, \frac{1}{2}, \frac{1}{3}\right)$  of the cubic

$$x = t, \quad y = \frac{1}{2} t^2, \quad z = \frac{1}{3} t^3.$$

2. Find the area of the region below the given curve and  $x$ -axis:

$$\left. \begin{array}{l} x = 3 \sin t \\ y = 4 \cos t \end{array} \right\} 0 \leq t \leq \frac{\pi}{2}.$$

3. Eliminate the parameter and sketch the curve

$$\begin{array}{l} x = 5 \cos t \\ y = 3 \sin t \end{array}$$

4. Find the area of the surface generated by revolving the given curve around the  $x$ -axis

$$\begin{aligned}x &= e^t \cos t \\y &= e^t \sin t\end{aligned} \quad 0 \leq t \leq \frac{\pi}{2}.$$

5. Compute:  $D_t[u(t) \cdot v(t)]$  where

$$\begin{aligned}u(t) &= \langle t, t^2 \rangle \\v(t) &= \langle t^2, -t \rangle\end{aligned}$$

6. Parametric equations of a moving point are

$$\begin{aligned}x &= 3 \cos 2t \\y &= 3 \sin 2t \\z &= 8t\end{aligned}$$

Find its velocity, speed and acceleration at  $t = \frac{7\pi}{8}$ .

7. Find area of triangle with vertices  $A(1, 3, -2)$ ,  $B(2, 4, 5)$  and  $C(-3, -2, 2)$ .

8. Parametrize the parabola  $y^2 = 4ax$  by expressing  $x$  and  $y$  as functions of the slope  $m$  of the tangent line at the point  $p(x, y)$  of the parabola.

9. A point moves with constant speed, so its velocity vector  $v$  satisfies the condition  $|v|^2 = c$ ,  $c$  is a constant. Show that the velocity and acceleration vectors of the point are always perpendicular to each other.

10. Let

$$\mathbf{a} = 2i - 3j + 5k$$

$$\mathbf{b} = 5i + 3j - 7k$$

Compute  $\text{comp}_{\mathbf{a}}(\mathbf{b})$  and  $\text{comp}_{\mathbf{b}}(\mathbf{a})$ .