CALCULUS III NAME ____________________

EXAM I Rec. Instr. ____________________

SPRING 1998 Rec. Time ____________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. For the parametric curve $x = t^3 + t, \quad y = t^5 + t$

a) find $\dfrac{dy}{dx}$ as a function of t.

b) find the equation of the line which is tangent to the curve at the point when $t = 1$.

c) set up the integral which gives the arc length of that piece of the curve obtained when $0 \leq t \leq 2$. Do not try to evaluate this integral.

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(25) 2. An object is moving in the plane according to $x = t^2 + 1$, $y = t^3 - 3t$ where t is the time. Find, as functions of t,

a) positon vector $\vec{r} =$

b) velocity vector $\vec{v} =$

c) speed $\dfrac{ds}{dt} =$

d) acceleration vector $\vec{a} =$

Now find

e) the angle between $\vec{v}$ and $\vec{a}$ when $t = 0$.

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(15) 3. An object is moving in the plane in such a way that its acceleration vector as a function of time is given by $\vec{a} = (\cos t)\vec{i} - \vec{j}$. Suppose you know that at time $t = 0$, the velocity vector is $\vec{v}(0) = \vec{j}$ and the position vector is $\vec{r}(0) = 2\vec{i}$. Find, as a function of t,

a) velocity vector $\vec{v}(t)$

b) position vector $\vec{r}(t)$

c) Now give the parametric equations for the motion.

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(20) 4. Given the points $P(1,2,3)$, $Q(2,2,0)$ and $R(1,3-1)$ in 3-space,

a) find the area of the triangle having P, Q and R as vertices.

b) find the angle of the triangle at the vertex P.

c) find the equation of the plane containing the points P, Q and R.

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(15) 5. Find parametric equations for the following lines:

a) Line through the points $P(2,1,3)$ and $Q(1,4,1)$.

b) Line through the point $P(3,0,-2)$ and perpendicular to the plane $2x - y + 4z + 1 = 0$.

c) Line of intersection of the planes $x + z = 1$ and $x - y = 1$.

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(10) 6. An object moves along a curve at a constant speed. Show that at each instant the velocity vector and the acceleration vector are perpendicular to each other.

Comment: This is homework problem 12.4 #35 or 13.4 #22. Note that it is not necessarily true that the acceleration vector is the zero vector. You might want to use that $\vec{v} \cdot \vec{v} = (\text{speed})^2$.