CALCULUS III NAME _____

EXAM I Rec. Instr. _____

FALL 1996 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(10) 1. For the parametric curve $x = t^3 + t, \quad y = t^5 - t$

a) Find $\dfrac{dy}{dx}$ as a function of t.

b) Find the equation of the tangent line to the curve when $t = 1$.

c) Give the definite integral which gives the arc length from $t = 1$ to $t = 3$. You do not need to evaluate the integral.

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(15) 2. A particle is moving in the plane. Its acceleration vector as a function of time is $\vec{a} = (\cos t)\vec{i} + \vec{j}$. Suppose that at $t = 0$ its velocity vector is $\vec{v}(0) = \vec{i} + \vec{j}$, and its position vector is $\vec{r}(0) = \vec{j}$. Find the velocity vector and the position vector as functions of t. Then write the parametric equations for the motion.

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(15) 3. A particle is moving in the plane along the curve $y = 2\sin x$. It is moving from left to right at a constant speed of 3 ft/sec.

a) Find a_T and a_N at the point $(x, 2\sin x)$.

b) Find the velocity vector and acceleration vector when the particle is at the point $\left(\frac{\pi}{2}, 2\right)$.

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(20) 4. A particle is moving in 3-space according to the parametric equations $x = \cos t\,,\ y = \sin t\,,\ z = t^2\,.$ Find, as functions of $t\,,$

a) position vector $\vec{r} =$

b) velocity vector $\vec{v} =$

c) acceleration vector $\vec{a} =$

d) speed $\dfrac{ds}{dt} =$

e) tangential component of acceleration $a_T =$

Now at time $t = \dfrac{\pi}{2}$ find the curvature $\kappa\,.$

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(25) 5. Given the vectors

$$\begin{aligned} \vec{a} &= 3\vec{i} + \vec{j} - 2\vec{k} \\ \vec{b} &= -\vec{i} + 2\vec{j} + \vec{k} \\ \vec{c} &= \vec{i} + 2\vec{j} - \vec{k} \end{aligned}$$

Find:

a) $|\vec{b} - \vec{a}|$

b) $\vec{a} \cdot \vec{b}$

c) the cosine of the angle between $\vec{a}$ and $\vec{b}$

d) $\vec{a} \times \vec{b}$

e) the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$

f) the volume of the parallelepiped determined by $\vec{a}$, $\vec{b}$ and $\vec{c}$

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(15) 6. Given the points $P(1,-1,2)$, $Q(3,1,5)$ and $R(1,2,3)$

Find

a) a unit vector in the direction of $\overrightarrow{PQ}$

b) parametric equations for the line through P and R

c) the equation of the plane containing the points P, Q and R