

Name (Please Print) _____

Recitation Instructor _____

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CALCULUS III

Exam I

September 28, 1995

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(10) 1. Find the arc length of the curve $x = \frac{1}{2}t^2$, $y = \frac{1}{3}t^3$, $0 \leq t \leq 2$.

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(20) 2. The motion of an object in 3space is given by the parametric equations

$$x = t^2, \quad y = \sin t, \quad z = \cos t$$

where t is the time in seconds and distances are measured in feet.

Determine the following:

a) The position vector for the motion $\mathbf{r}(t) =$

b) The velocity vector $\mathbf{v}(t) =$

c) The acceleration vector $\mathbf{a}(t) =$

d) The speed $\frac{ds}{dt} =$

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- (20) 3. An object is moving in the plane in such a way that its acceleration vector is $\mathbf{a}(t) = (\cos t)\mathbf{j}$. If the initial velocity vector is $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$ and the initial position vector is $\mathbf{r}(0) = 2\mathbf{i}$, find the velocity vector $\mathbf{v}(t)$ as a function of time t and then the position vector $\mathbf{r}(t)$. What are the parametric equations that give the motion?

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(20) 4. Given the points $P(1, 1, 0)$, $Q(1, 0, 1)$, $R(0, 1, 1)$ in 3space.

a) Find the area of the triangle having P , Q , and R as vertices.

b) Find the volume of the parallelepiped with adjacent edges \vec{OP} , \vec{OQ} , and \vec{OR} .

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(15) 5. Find the equation for each of the following planes:

a) The plane through $P(2, 3, -4)$ with normal vector $\mathbf{n} = 7\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$.

b) The plane through the points $A(1, 0, -1)$, $B(3, 3, 2)$, $C(4, 5, -1)$

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(15) 6. A particle is moving in the plane along the curve $y = x^2$ from left to right. It is moving at a constant speed of 2 ft/sec.

a) Find a_T and a_N at (x, x^2) .

b) Find the velocity vector and the acceleration vector when the particle is at the point $(0, 0)$.