CALCULUS III NAME ______________________

EXAM I Rec. Instr. ______________________

FALL 2001 Rec. Time ______________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1) An object is moving in 3-space according to the parametric equations $x = t$, $y = \dfrac{1}{2}t^2$, $z = \sin t$. Find as functions of the time t

a) the position vector $\vec{r} =$

b) the velocity vector $\vec{v} =$

c) the acceleration vector $\vec{a} =$

d) the speed $\dfrac{ds}{dt} =$

e) the tangential component of acceleration $a_T =$

For items f) and g), at $t = \dfrac{\pi}{2}$ find

f) the curvature κ

g) the normal component of acceleration $a_N =$

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(15) 2) An object is moving in the plane along the curve $y = 2\ln x$, $x > 0$, from left to right. It is moving at a constant speed of 5 ft/sec.

a) Find a_T and a_N when the object is at the point $(x, 2\ln x)$.

b) Find the velocity vector and the acceleration vector when the object is at the point $(1, 0)$.

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(15) 3) A object is moving in 3-space in such a way that its acceleration vector as a function of time is $\vec{a} = (\sin t)\vec{i} - \vec{k}$. Suppose you know that at time $t = 0$ the velocity vector is $\vec{v}(0) = \vec{j}$ and the position vector is $\vec{r}(0) = 10\vec{k}$. Find the velocity vector and position vector as functions of the time t and then give the parametric equations for the motion.

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(25) 4) Given the three points $A(1,0,1)$, $B(2,1,1)$ and $C(-1,0,3)$. Let ΔABC denote the triangle having A, B, C as its vertices.

a) Find the angle at the vertex A of the triangle ΔABC. Give the answer in degrees.

b) Find the area of the triangle ΔABC.

c) Give the equation of the plane containing the triangle ΔABC.

d) Give the parametric equations for the line through the point C which is perpendicular to the triangle ΔABC.

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(25) 5) Let $f(x, y) = 2x^3 - 3x^2y + 3y^2 + 9y$.

a) Calculate all 1st and 2nd partial derivatives of $f(x, y)$.

b) Find the equation of tangent plane to the surface $z = f(x, y)$ at the point $(1, 0, 2)$.

c) Find all points on the surface $z = f(x, y)$ where the tangent plane is horizontal.