CALCULUS III NAME ____________________

EXAM Rec. Instr. ____________________

SPRING 1997 Rec. Time ____________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(40) 1. A particle is moving in 3-space according to the position vector function
$\vec{r} = \cos t\,\vec{i} + \sin t\,\vec{j} + \sin t\,\vec{k}$.

Find, at time t,

a) velocity vector $\vec{v}$ =

b) acceleration vector $\vec{a}$ =

c) the speed $\dfrac{ds}{dt}$ =

d) the tangential component of acceleration a_T =

e) the curvature κ =

f) the normal component of acceleration a_N =

g) Now at $t = \dfrac{\pi}{4}$ secs find the angle between the velocity vector and the acceleration vector.

(20) 2. Let $f(x, y) = x^3 + 2xy^2 + y^3$.

a) find the gradient vector field of f .

$\bigtriangledown f =$

b) find the directional derivative of $f(x, y)$ at $(x, y) = (1, 2)$ in the direction of the vector $\vec{a} = 2\vec{\imath} - 3\vec{\jmath}$.

c) find the value of the largest directional derivative of $f(x, y)$ at $(x, y) = (1, 2)$.

d) find a unit vector which points in the direction which gives the largest directional derivative of $f(x, y)$ at $(x, y) = (1, 2)$.

(15) 3. A quantity Q depends upon x and y according to $Q = y^3 e^{(x/y)}$. Both x and y are changing with time and at a certain instant you know that $x = 4$, $y = 2$, $\dfrac{dx}{dt} = 1$, $\dfrac{dy}{dt} = -2$. Use the chain rule to find $\dfrac{dQ}{dt}$ at this instant.

(25) 4. Find the equation for each of the following planes.

a) plane containing $(2,1,-1)$ and perpendicular to the line $x = 1+3t$, $y = 2t$, $z = 2-2t$

b) plane containing the points $(1,2,3)$, $(3,3,1)$ and $(1,3,4)$.

c) the tangent plane to the surface $x^2y + y^2z + z^2 = 4$ at the point $(2,1,-1)$

(20) 5. Let $f(x, y) = 6xy - 3xy^2 - x^3$. Find all local maximum points, local minimum points and saddle points. That is, find and classify all critical points.

(15) 6. A mass distribution occupies the region which is above the cone $z = \sqrt{x^2 + y^2}$ and under the plane $z = 4$. The mass density function is $\delta(x, y, z) = z^2$. Use a triple integral in spherical coordinates to calculate the total mass.

(15) 7. Find the surface area of that part of the paraboloid $z = x^2 + y^2$ which is between the cylinders $x^2 + y^2 = 2$ and $x^2 + y^2 = 6$.

(15) 8. A solid body with mass density function $\delta(x, y, z) = xz$ occupies the region in the first octant bounded by the surfaces $z = y$, $z = 4$, $y = x^2$, $y = 1$, and $x = 0$. Find the total mass.

(15) 9. The force field $\vec{F} = xy\,\vec{i} - x^2\vec{j}$ acts on an object as it is moved in the plane. Find the work done by $\underline{F}$ as the object moves from $(0,0)$ to $(4,1)$ along the curve which is the parabola $y = x^2$ from $(0,0)$ to $(1,1)$ followed by the line segment from $(1,1)$ to $(4,1)$.

(10) 10. Show that the force field $\vec{F} = \left(\left(\frac{x}{y}\right) + 1\right)\vec{i} + \left(-\frac{1}{2}\left(\frac{x^2}{y^2}\right) + 2y\right)\vec{j}$ is conservative in the upper half plane $(y > 0)$ by finding a potential function for $\vec{F}$. Then use this potential function to calculate the work done by $\vec{F}$ when it acts on a particle which is moved from $(0, 1)$ to $(4, 2)$ in the upper half plane.

(10) 11. Use Green's theorem to evaluate the line integral $\int_C 3x^2y\,dx + (x^3 + 4x)\,dy$ where C is the circle $(x-5)^2 + (y+1)^2 = 25$ directed counter-clockwise.