CALCULUS III NAME ____________________

EXAM Rec. Instr. ____________________

SPRING 1996 Rec. Time ____________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(30) 1. A particle is moving in 3-space according to the position vector function $\mathbf{r} = \cos t\ \mathbf{i} + \sin t\ \mathbf{j} + \sin t\ \mathbf{k}$.

Find, at time t,

a) velocity vector $\mathbf{v}$ =

b) acceleration vector $\mathbf{a}$ =

c) the speed $\frac{ds}{dt}$ =

d) the tangential component of acceleration a_T =

e) the curvature κ =

f) the normal component of acceleration a_N =

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(25) 2. Find the equation for each of the following planes:

a) The plane containing the points $P_1(1,2,-1), P_2(2,1,3), P_3(1,1,1)$

b) The tangent plane to $xyz = 6$ at the point $(1,2,3)$

c) The tangent plane to the graph of $f(x,y) = 3xy^2 - 2y^3 - 2x^3$ at the point $(2,1,-12)$

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(20) 3. Let $f(x,y) = \dfrac{ye^{xy}}{300}$.

a) Find the gradient vector field of f.

$\bigtriangledown f =$

b) Find the directional derivative of f at $(2,3)$ in the direction of the vector $\mathbf{a} = \mathbf{i} - 2\,\mathbf{j}$.

c) Find a unit vector which points in the direction which gives the largest directional derivative for f at $(2,3)$.

d) Find the value of the largest directional derivative for f at $(2,3)$.

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(20) 4. Given the three vectors $\mathbf{a} = \langle 1, 1, 1 \rangle, \mathbf{b} = \langle 2, -4, 1 \rangle, \mathbf{c} = \langle 1, 2, 2 \rangle$. Find

a) the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$

b) a nonzero vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$

c) the area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$

d) the volume of the parallelepiped determined by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$

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(20) 5. Let $f(x,y) = x^3 - xy^3 + 3xy$. Find all local maximum points, local minimum points, and saddle points.

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(20) 6. Let D be the 3-dimensional region in the first octant enclosed by the surfaces $z = y$, $z = 2$, $y = x^2$, $y = 1$, and $x = 0$.

(a) use a triple integral to find the volume of D.

(b) If a solid body occupies D with a mass density function of $\delta(x, y, z) = yz$, find the total mass.

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(20) 7. a) Use a double integral to find the area of the region in the xy-plane which is enclosed by the curves $x = y^2 - 4y$ and $x + y = 4$.

(b) Find the <u>surface</u> <u>area</u> of that part of the surface $z = x^2 + y^2$ which is above the region in the first quadrant of the xy-plane that is bounded by the curves $y = x$, $x^2 + y^2 = 4$, and $y = 0$. Use polar coordinates to evaluate the double integral.

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(15) 8. Use a triple integral in spherical coordinates to calculate the total mass of a mass distribution which occupies the region bounded by the plane $z = 2$ and the cone $\phi = \cos^{-1}(\frac{1}{3})$ if the mass density function is $\delta(x, y, z) = z$.

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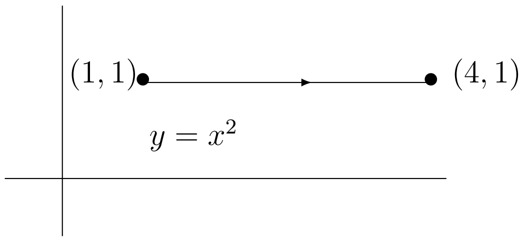
(15) 9. The force field $\mathbf{F} = xy\,\mathbf{i} - x^2\,\mathbf{j}$ acts on an object as it is moved in the plane. Find the work done by $\mathbf{F}$ as the object moves from $(0,0)$ to $(4,1)$ along the curve which is drawn.



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(15) 10. Show that the force field $\mathbf{F} = (-2x + (1+y)^{-1})\,\mathbf{i} + (-x(1+y)^{-2} + 2)\mathbf{j}$ is conservative in the region $y \neq -1$ by finding a potential function for $\mathbf{F}$. Now use this potential function to calculate the work done by $\mathbf{F}$ if it acts on an object which is moved along any piecewise smooth curve in the upper half plane from $(2, 0)$ to $(4, 3)$.