CALCULUS III NAME ____________________

EXAM Rec. Instr. ____________________

FALL 1998 Rec. Time ____________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. An object is moving in 3-space in such a way that its acceleration vector as a function of time is $\vec{a} = (\cos t)\vec{i} - (\sin t)\vec{j}$. Suppose you know that at time $t = 0$, the velocity vector is $\vec{v}(0) = \vec{i} + \vec{k}$ and the position vector is $\vec{r}(0) = \vec{j}$. Find the velocity vector and the position vector as functions of t, and then give the parametric equations of the motion.

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(15) 2. A particle is moving in the plane along the curve $y = 2x^2 - x$ in the direction of increasing x. It is moving at a constant speed of 3 ft/sec.

a) Find a_T and a_N when the particle is at $(x, 2x^2 - x)$.

b) Find the velocity vector and the acceleration vector when the particle is at $(0, 0)$.

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(15) 3. A quantity Q depends upon x and y according to $Q = y^2 e^{x^2 y}$. Suppose x and y are changing with the time t and you know that at a certain instant $x = 2$, $y = -1$, $\frac{dx}{dt} = 3$, $\frac{dy}{dt} = -3$. Use the chain rule to find $\frac{dQ}{dt}$ at this instant.

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(15) 4. Let $f(x, y) = x^3 - xy^3 + 3xy$.

a) If you are moving along a straight line throught the point $(x, y) = (1, 2)$ toward the point $(x, y) = (4, 1)$ what is the instantaneous rate of change of $f(x, y)$ per unit distance that you will observe at $(1, 2)$?

b) If you are at $(x, y) = (1, 2)$, in which direction should you go if you want $f(x, y)$ to increase most rapidly?

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(20) 5. Find the equation of the tangent plane to each of the following surfaces at the specified point.

a) to the graph of $f(x,y) = x\sqrt{x^2+y^2}$ at $(3,4,15)$

b) to $xy^3 + yz^3 + zx^3 = 5$ at $(1,2,-1)$

c) to the parametrized surface $x = s^2 + t\,,\ y = t^2 + s\,,\ z = s^3$ at the point obtained when $s = 1$ and $t = 2\,.$

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(20) 6. Find and classify the critical points of $f(x,y) = x^3 - xy^3 + 3xy$.

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(15) 7. Use the method of Lagrange multipliers to find the largest value and the smallest value of $f(x, y, z) = xz + y^2$ on the sphere $x^2 + y^2 + z^2 = 4$.

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(15) 8. Find the volume of the 3-D region enclosed by the surfaces $y = x^2$, $y = 4$, $z = 5 + x$, $z = 2$.

(15) 9. A mass distribution occupies the region which is above $z = x^2 + y^2$ and under $z = 4$. If the mass density function is $\delta(x, y, z) = 2z$ units of mass/unit volume, calculate the total mass. Use cylindrical coordinates to evaluate the triple integral.

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(10) 10. Show that the force field

$$\vec{F} = \left(2x + \frac{2x}{(x^2+y^2)^2}\right)\vec{\imath} + \left(1 + \frac{2y}{(x^2+y^2)^2}\right)\vec{\jmath}$$

is conservative in the region $(x, y) \neq (0, 0)$ by finding a potential function for it. Now use this potential function to calculate the work done by $\vec{F}$ as it acts on an object which moves from $(2, 1)$ to $(3, 4)$ along any path which does not pass through $(0, 0)$.

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(10) 11. Use Green's theorem to evaluate the line integral $\int_C (x^2 + 12xy)\, dx + (12x^2 + y^2)\, dy$ where C is the triangle with vertices $(0,0), (1,0), (0,1)$ directed counterclockwise.

(10) 12. Evaluate the surface integral $\int\int_S z^2 dS$ where S is the parametrized surface $x = 4\cos t\,,\ y = 4\sin t\,,\ z = s\,,\ 0 \le t \le 2\pi\,,\ 0 \le s \le 3\,.$

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(15) 13. Verify the divergence theorem (page 940) for the special case where the vector field is $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ and the surface S is the sphere $x^2 + y^2 + z^2 = 4$ by direct calculation of both the surface integral and the triple integral.

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(10) 14. Let $\vec{F}$ be the vector field $\vec{F} = z\vec{i} + xyz\vec{j} - x\vec{k}$.

a) $curl\vec{F} = \nabla \times \vec{F} =$

b) Use Stokes' theorem to evaluate $\int_C \vec{F} \cdot \vec{T}\, ds$ where C is the circle $x^2 + z^2 = 9$, $y = 3$ and S is the surface $x^2 + z^2 \leq 9$, $y = 3$ with normal vector $\vec{n} = \vec{j}$.

First, $curl\vec{F} \cdot \vec{n} =$

then $\int\int_S (curl\vec{F} \cdot \vec{n}) dS =$

so $\int_C (\vec{F} \cdot \vec{T}) ds =$