

Name _____

Rec. Instr. _____

CALCULUS III

Final Exam

Fall 1995

(35) 1. The motion of an object in 3-space is given by $x = t^2$, $y = t$, $z = \sin t$ where t is the time measured in seconds and distances are measured in feet. Determine the following:

(a) The position vector for the motion $\mathbf{r}(t) =$

(b) The velocity vector $\mathbf{v}(t) =$

(c) The acceleration vector $\mathbf{a}(t) =$

(d) The speed $\frac{ds}{dt} =$

(e) The tangential component of acceleration $a_T =$

(f) The curvature at $t = \pi$ seconds

(g) The normal component of acceleration a_N at $t = \pi$ seconds.

Name _____

Rec. Instr. _____

(15) 2. A particle is moving in the plane along the curve $y = \sin x$ from left to right. It is moving at a constant speed of 3 ft/sec.

(a) Find a_T and a_N at the point $(x, \sin x)$.

(b) Find the velocity vector and the acceleration vector when the particle is at the point $\left(\frac{\pi}{2}, 1\right)$.

Name _____

Rec. Instr. _____

(25) 3. Given the points $P(2, 1, 0)$, $Q(1, 1, 1)$, $R(0, 3, 1)$ in 3-space

(a) Find the area of the triangle having P , Q , and R as vertices.

(b) Find the volume of the parallelepiped with adjacent edges \overrightarrow{OP} , \overrightarrow{OQ} , and \overrightarrow{OR} .

(c) Find the equation of the plane containing the points P , Q , and R .

Name _____

Rec. Instr. _____

(15) 4. A quantity Q depends upon x and y according to $Q = x\sqrt{y^2 + x}$. Suppose x and y are changing with time and at $t = 4$ seconds we know that

$$x = 5, \quad y = 2, \quad \frac{dx}{dt} = -6, \quad \text{and} \quad \frac{dy}{dt} = 3.$$

Use the **Chain Rule** to find $\frac{dQ}{dt}$ at $t = 4$ seconds.

Name _____

Rec. Instr. _____

(25) 5. Let $f(x, y) = xe^{xy}$.

(a) Find the directional derivative of f at $(x, y) = (-1, 2)$ in the direction of the vector $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$.

(b) Find a unit vector which points in the direction in which f is increasing most rapidly at $(-1, 2)$ and then find the value of this largest directional derivative of $f(x, y)$ at $(-1, 2)$.

(c) Find the equation of the tangent plane to the graph of $f(x, y)$ at $(-1, 2, -e^{-2})$.

Name _____

Rec. Instr. _____

- (15) 6. Use the **method of Lagrangian Multipliers** to find the maximum value and the minimum value of $f(x, y) = x^2 - 2x - y^2$ on the circle $x^2 + y^2 = 4$.

Name _____

Rec. Instr. _____

- (20)** 7. Let $f(x, y) = y^3 + 2x^2y - 8xy + 6y$. Find all critical points and determine whether each critical point gives a local maximum, a local minimum, or a saddle point.

Name _____

Rec. Instr. _____

(10) 8. Find the surface area of the part of $z = 16 - x^2 - y^2$ which is between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

(10) 9. Find the volume of the 3-dimensional region in the 1st octant which is enclosed by the surfaces $y = x^2$, $y = 2x$, $z = 0$, and $z = x^2 + y^2$.

Name _____

Rec. Instr. _____

- (15) 10. A mass distribution occupies the region which is between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 9$. If the mass density function is $\delta(x, y, z) = z^2$ units of mass/unit volume find the total mass.

- (15) 11. The force field $\mathbf{F} = -y\mathbf{i} + (x + 2)\mathbf{j}$ acts on a particle as it moves from $(1, 0)$ to $(0, 2)$ along the following path: first along the circle $x^2 + y^2 = 1$ in the 1st quadrant from $(1, 0)$ to $(0, 1)$; second along the segment of the y -axis from $(0, 1)$ to $(0, 2)$. Calculate the work done by \mathbf{F} .