Name (Please Print) _____ Rec. Instr. _____

Your Signature _____ Class Time _____

ANALYTIC GEOMETRY AND CALCULUS III

Final Exam

December 15, 1994

The point value of each problem is indicated in the left margin. You must show all of your work for full credit. Points will be deducted for faulty reasoning, for sloppy notation, and for failure to simplify answers, even if your answer is correct. You may use a calculator, your class notes, and any reference material. Explicitly cite, in some manner, any published formulae you use.

(24) 1. Reparametrize the circular helix

$$\mathbf{r}(t) = (5\cos t, 5\sin t, 12t), \quad t \geq 0$$

by arclength.

Answer $\mathbf{r}_1(s) =$ _____

(24) 2. Find a vector equation for the line of intersection of the planes:

$$\begin{aligned} 2x + 3y - z &= -3, \quad \text{and} \\ 4x + 5y + z &= 1. \end{aligned}$$

Answer $(\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_) \times [\mathbf{r} - (\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_)] = \mathbf{0}$

(24) 3. Let $w = \ln(u^2 + v^2)$, $u = x - y$ and $v = x + y$.
Find $\dfrac{\partial w}{\partial y}$ at $(x, y) = (1, 1)$.

Answer $\dfrac{\partial w}{\partial y}(1, 1) =$ _____

(24) 4. Find an equation for the tangent plane at $P = (3, 2, 19)$ to the surface $z = x^3 - y^3$.

Answer _____

(24) 5. Locate and classify the critical points of

$$f(x, y) = 3x - x^3 - 3xy^2.$$

Answers: local minima at _____

local maxima at _____

saddle points at _____

(24) 6. Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{1}{1+x^2+y^2}\, dx\, dy$.

Answer _____

(28) 7. What is the volume of the solid bounded above by the paraboloid $z = 8 - (x^2 + y^2)$ and below by the paraboloid $z = x^2 + y^2$?

Answer _____

(28) 8. What is the surface area of the portion of the paraboloid $z = 9 - (x^2 + y^2)$ that lies above the plane $z = 5$?

Answer _____