CALCULUS III — NAME ______________________

FINAL EXAM — Rec. Instr. ______________________

FALL 2001 — Rec. Time ______________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK

(15) 1. An object is moving in 3-space in such a way that its acceleration vector as a function of the time t is $\vec{a} = (\cos t)\vec{i} + (\sin t)\vec{j} + \vec{k}$. At time $t = 0$ its velocity vector is $\vec{v}(0) = -\vec{j}$. Find the tangential component of acceleration a_T, the curvature κ, and the normal component of acceleration a_N as functions of t.

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(20) 2. Let $f(x, y) = x^2 y + \frac{1}{2} y^2$.

a) Suppose you are moving along a straight line through the point $(x, y) = (1, 2)$ toward the point $(x, y) = (2, 0)$. What is the instantaneous rate of change of $f(x, y)$ per unit distance that you will observe at $(x, y) = (1, 2)$?

b) What is the largest possible directional derivative of $f(x, y)$ at $(x, y) = (1, 2)$? Find a unit vector which points in the direction which gives this maximal directional derivative.

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(25) 3. Given that z is a function of x and y, $z = f(x, y)$ and that $x = \frac{u}{v}$, $y = u^2 - v$ with $u > 0$. This determines z as a function of u and v. Suppose you know that at $(x, y) = (1, 2)$, $\frac{\partial f}{\partial x}(1, 2) = 2$ and $\frac{\partial f}{\partial y}(1, 2) = 3$. Use the chain rule to fine $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ when $(x, y) = (1, 2)$.

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(20) 4. Find the equation of the tangent plane to the given surface at the specified point.

a) To $xz\sqrt{y^2+2x} = 80$ at $(8, 3, 2)$.

b) To the parametric surface $x = s + t$, $y = s - t$, $z = s^2$ at the point obtained where $s = 1$ and $t = 1$.

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(25) 5. Find and classify the critical points of $f(x, y) = x^3 - xy^3 + 3xy$.

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(15) 6. Calculate the volume of the 3D-region enclosed by the surfaces $y = x^2$, $y = 8 - x^2$, $z = 6 + x$ and $z = 2x$.

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(15) 7. A mass distribution occupies the region which is inside $x^2 + y^2 + z^2 = 4$ and above $z = \sqrt{x^2 + y^2}$. If the mass density function is $\delta(x, y, z) = z^2$ units of mass/unit volume, use a triple integral in spherical coordinates to calculate the total mass.

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(15) 8. Evaluate the surface integral $\int\int_X (x^2 - y^2)\, dS$ where S is the parametrized surface $x = s + t$, $y = s - t$, $z = s^2$, $0 \le s \le 2$, $0 \le t \le 3$.

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(15) 9. The force field $\vec{F} = y\vec{i} - x\vec{j}$ acts on an object as it moves in the plane. Calculate the work done by $\vec{F}$ as the object moves from $(0,0)$ to $\left(0, \sqrt{2}\right)$ by first going along $y = x^2$ from $(0,0)$ to $(1,1)$ and then along the first quadrant arc of the circle $x^2 + y^2 = 2$ from $(1,1)$ to $\left(0, \sqrt{2}\right)$.

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(15) 10. Use Green's theorem to evaluate

$$\int_C (4x^3y^4 - x^2y)\,dx + (y^6 + 4x^4y^3)\,dy$$

where C is the circle $x^2 + y^2 = 9$ directed counterclockwise.

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(15) 11. Use the divergence theorem to evaluate $\int\int_S \vec{F} \cdot \vec{n}\, dS$ where $\vec{F} = xz^2\vec{i} + y\vec{j} + z\vec{k}$, S is the sphere $x^2 + y^2 + z^2 = 1$ with $\vec{n}$ the outward pointing unit normal.

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(15) 12. Let S be that part of the plane $x+y+z=2$ which is in the first octant oriented with the upward pointing normal and let C be the positively oriented boundary of S. If $\vec{F}=(xz)\vec{i}+x\vec{j}+y\vec{k}$ use Stokes' theorem to evaluate

$$\int_C \vec{F}\cdot\vec{T}\,ds=\int_C xz\,dx+x\,dy+y\,dz$$