

The Happy and Joyous (and Joyous and Happy) Practice Midterm

MA501, Fall 2004

The following instructions were used for the practice test for an undergrad course in Louisville, but they were too much fun to throw out. Besides, they still pretty much apply. Enjoy.

You will not be allowed to use any type of calculator whatsoever, you will not be allowed to have any other notes, the test will be closed book, and there is no escape. The actual test will be graded **in red ink!** There will be no mercy for the weak. Mathematics is cumulative. Deal with it. What you don't know **will** hurt you. You need to be able to make simple and/or standard simplifications. In order to get credit or partial credit, your work must make sense.

I strongly suggest that you take this practice test under the conditions of the actual test! (The only exception being that since this test is longer than the actual test, it makes sense for you to take it in more than one sitting.)

Before the real practice test below, you should go find some dot products, cross products, magnitudes, determinants, gradients, directional derivatives, divergences, curls, and Laplacians of the appropriate objects. I'm not going to put questions down for this stuff because I assume that

1. you'll know this stuff in your sleep by the time the midterm comes, and
2. you can go find this kind of exercise on your own.

Note that **none** of the things just mentioned will be available on the cheat sheet. In other words, I view them as the basic "vocabulary" that you should memorize. Other things related to the stuff above which should be memorized:

$$\vec{u} \bullet \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cos \theta, \quad \|\vec{u} \times \vec{v}\| = \|\vec{u}\| \cdot \|\vec{v}\| \sin \theta$$

Other basic properties can be found in the text, and also in the vector calculus notes I put on the web. (Certainly, for example, I expect you to calculate directional derivatives in the simple way!)

1. Find the scalar projection of $(3, 7)$ in the direction of $(1, 2)$.
2. Find the area of the parallelogram spanned by $(2, 3)$ and $(1, 4)$.
3. Find the volume of the parallelepiped spanned by $(1, 2, 0)$, $(0, 1, 2)$ and $(1, 1, 1)$

4. Find an equation for the plane through $(1, 2, 3)$, $(1, 3, 2)$ and $(2, 2, 2)$.
5. Let $\vec{r}(t) := (3 \cos t, 4 \sin t, t^2)$, and use the formulas for curves that are posted on my website for this problem. (Note that you should know velocity and acceleration without that sheet!)
 - (a) Find the unit tangent vector at time t .
 - (b) Find the curvature at time t .
 - (c) Find the normal and tangential components of acceleration at time t .
6. For the function $f(x, y) := x^2 + 2xy + 3y^2$ if you are at the point $(1, 2)$ and can move a teenie-tiny bit to increase your function, then in which direction should you move?
7. Suppose that you want to minimize the function

$$x^2 z^3 \sin(y^2 + z^2)$$

subject to the constraints:

$$x^2 + y^2 + z^2 = 100 \quad \text{and} \quad xy = 1 .$$

Then write down the system of equations you need to solve if you want to use Lagrange multipliers to find the answer. (I expect that the equations themselves will not be solvable without a computer, so don't try to solve them.)

8. Find the minimum and maximum of $3x + 4y$ subject to the constraint $x^2 + y^2 = 25$. (Use Lagrange Multipliers for this problem!!!)
9. Let C be the subset of the graph of $y = x^2$ with $2 \leq x \leq 3$ oriented so that $(2, 4)$ is the first point and $(3, 9)$ is the last point. Find the following line integrals:

$$\int_C (xy, y) \bullet d\vec{r}, \quad \int_C (-\pi y \sin(\pi xy), -\pi x \sin(\pi xy)) \bullet d\vec{r},$$

$$\int_C (2xe^{(x^2+y)}, e^{(x^2+y)}) \bullet d\vec{r} .$$

10. Integrate the function $e^{(x^2+y^2)}$ over the disk $x^2 + y^2 \leq 4$.

11. Let

$$S := \{ (x, y, z) : z = x^2 + y^2, x \geq 0, z \leq 9 \}$$

and let $\vec{F}(x, y, z) := (x + y, y - z, x - z)$.

- (a) Express S as a parametric surface.
- (b) Write out an explicit formula for

$$I_1 := \int \int_S (\nabla \times \vec{F}) \bullet \vec{n} \, dA$$

in terms of the parametrization you just found. (Note that this means that all bounds of integration must be determined.)

- (c) Find I_1 .
- (d) Write out an explicit formula for

$$I_2 := \int_{\partial S} \vec{F} \bullet d\vec{r}.$$

Choose your orientation of ∂S so that your value for I_2 is the same as the value of I_1 .

- (e) Compute it and check that it is equal to what you found in part (c).
- (f) Sing the Happy Helmet Song and do the Happy Dance!

12. Let

$$V := \{ (x, y, z) : x + 2y + 3z \leq 6 \} \cap O_1$$

where O_1 is the first octant, and let $\vec{F}(x, y, z) := (2x + 3y^2, y + z, 2)$.

- (a) Find

$$I_1 := \int \int_{\partial V} \vec{F} \bullet \vec{n} \, dA$$

where \vec{n} is the outward unit normal, without using the divergence theorem. (In other words, do the surface integrals.)

- (b) Now use the divergence theorem to find I_1 by transforming it into a triple integral over a volume.