Definitions

Let S be an arbitrary subset of $\mathbb{R}^n$. x is an *accumulation point* of S if for every $r > 0$,

$$B_r(x) \cap S \setminus \{x\} \neq \phi \ .$$

Equivalently, x is a *limit point* of S if there exists a sequence $\{x_n\} \subset S \setminus \{x\}$ such that $x_n \to x$.

The set of accumulation points of S is denoted by S' and $\bar{S} := S \cup S'$.

S is *closed* if $S' \subset S$.

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x is an *interior point* of S if there exists an $r > 0$ such that $B_r(x) \subset S$.

The set of interior points of S is denoted by $int(S)$ or S^o.

S is *open* if $S = S^o$.

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Second Derivative Test in $\mathbb{R}^2$.

Assume that $\nabla f(x_0, y_0) = (0, 0)$. Let

$$D(x, y) := f_{xx}(x, y) f_{yy}(x, y) - f_{xy}^2(x, y) \ .$$

- If $D(x_0, y_0) > 0$ and $f_{xx}(x_0, y_0) > 0$, then (x_0, y_0) is a local minimum.
- If $D(x_0, y_0) > 0$ and $f_{xx}(x_0, y_0) < 0$, then (x_0, y_0) is a local maximum.
- If $D(x_0, y_0) < 0$, then (x_0, y_0) is a saddle point.
- If $D(x_0, y_0) = 0$, then the test is inconclusive.