Formulas for Curves in 2 and 3 Dimensions

$$\vec{r}(t) = (x(t), y(t), z(t))$$

In 2-D:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} \qquad \kappa = \frac{|y''|}{(1+(y')^2)^{3/2}}$$

In general:

Velocity	$\vec{v}(t) := \vec{r}\,'(t)$
Acceleration	$\vec{a}(t) := \vec{v}\,'(t) = \vec{r}\,''(t)$
Arclength	$s(t) := \displaystyle\int_a^t ||\vec{r}\,'(u)||\ du$ (measuring from a to t)
Speed	$\dfrac{ds}{dt} = ||\vec{r}\,'(t)|| = ||\vec{v}(t)||$
Unit Tangent Vector	$\vec{T}(t) := \dfrac{\vec{v}(t)}{||\vec{v}(t)||} = \dfrac{\vec{r}\,'(t)}{||\vec{r}\,'(t)||}$
Curvature Vector	$\dfrac{d\vec{T}}{ds} = \dfrac{\vec{T}\,'(t)}{||\vec{v}(t)||} = \dfrac{\vec{T}\,'(t)}{||\vec{r}\,'(t)||}$
Curvature	$\kappa := \left\|\left\|\dfrac{d\vec{T}}{ds}\right\|\right\|$
Principle Unit Normal	$\vec{N}(t) := \dfrac{1}{\kappa}\dfrac{d\vec{T}}{ds} = \dfrac{\vec{T}\,'(t)}{\left\|\left\|\vec{T}\,'(t)\right\|\right\|}$
Binormal	$\vec{B}(t) := \vec{T}(t) \times \vec{N}(t)$

$$\vec{a}(t) = \frac{d^2s}{dt^2}\vec{T} + \left(\frac{ds}{dt}\right)^2 \kappa\vec{N} = a_T\vec{T} + a_N\vec{N}$$

$$a_T = \frac{\vec{r}\,' \bullet \vec{r}\,''}{||\vec{r}\,'||} \qquad a_N = \frac{||\vec{r}\,' \times \vec{r}\,''||}{||\vec{r}\,'||} \qquad \kappa = \frac{||\vec{r}\,' \times \vec{r}\,''||}{||\vec{r}\,'||^3}$$