

Abstracts of contributed talks

Ryan Berndt, Ohio State University, *A_p weights on domains and an extrapolation theorem.*
We define a class of A_p weights that give rise to a maximal function (defined on a domain Ω) bounded on $L^p(\Omega)$, $1 < p < \infty$. From this we obtain a Rubio de Francia type extrapolation theorem for operators defined on domains.

Ivan Blank, Worcester Polytechnic Institute, *The Hele-Shaw problem as a "Mesa" limit of Stefan problems: Existence, uniqueness, and regularity of the free boundary.*

We study a Hele-Shaw problem as a "Mesa" type limit of one phase Stefan problems in exterior domains. We study the convergence, determine some of the qualitative properties of the unique limiting solution, and prove regularity of the free boundary of this limit under very general conditions on the initial data. Indeed our results handle changes in topology and multiple injection slots. This project is joint work with Marianne Korten and Charles Moore.

Weidong Chen, Kansas State University, *An efficient method for band-limited extrapolation by regularization*

In this presentation, I will discuss the problem that I will attack —an efficient algorithm for solving a band-limited extrapolation. A regularized spectral estimation formula and a regularized iterative algorithm for band-limited extrapolation are presented. The ill-posedness is taken into account. First the Fredholm equation is regularized. Then it is transformed to a differential equation in the case where the time interval is \mathbb{R} . A fast algorithm to solve the differential equation is given by the finite difference and a regularized spectral estimation formula is obtained. Then a regularized iterative extrapolation algorithm is introduced and compared with the Papoulis and Gerchberg algorithm.

Maria Angeles Alfonseca Cubero, Kansas State University, *An almost- orthogonality principle for directional maximal functions.*

Given a set $\Omega \subset [0, \frac{\pi}{4}]$, let us consider any ordered subset $\Omega_0 = \{\theta_1 > \theta_2 > \dots > \theta_j > \dots\}$ of Ω . We decompose Ω in several disjoint blocks, $\Omega_j, j \geq 1$, separated by the elements of Ω_0 . For each $j \geq 0$, we define the maximal operator associated to Ω_j as

$$M_{\Omega_j}f(x) = \sup_{x \in R \in B_j} \frac{1}{|R|} \int_R |f(y)| dy,$$

where B_j is the basis of all rectangles forming an angle θ_j with the x -axis.

We look for a relation between the norm of $M_{\Omega}f(x, y) = \sup_{j \geq 1} M_{\Omega_j}f(x, y)$ as an operator in $L^2(\mathbb{R}^2)$, and the norms of the M_{Ω_j} . By means of a covering lemma, we obtain an almost-orthogonality principle in L^2 for the family $\{M_{\Omega_j}\}$. In order to extend this result to other values of p , we need to introduce a Littlewood-Paley decomposition, and the associated square function. As a corollary of these results, we are able to give a very simple proof of a recent result by Katz. Also, the almost-orthogonality principle allows us to extend some other results about directional maximal functions.

This is joint work with Fernando Soria and Ana Vargas.

Xiang Fang, Kansas State University, *Operator theory on the Hardy space over the polydisc.*

For the Hardy space over the unit disc in the complex plane, one can ask easy questions from the viewpoint of operator theory, and get nontrivial, often satisfactory, answers from

function theory. This is probably best exemplified by the Beurling's theorem, which describes the invariant subspaces by inner functions.

Let us move on to the polydisc. Now it is still easy to ask good questions, but the satisfactory answers are, in general, very hard to get. In fact, we still don't know much about the lattice of invariant subspaces in this case.

In our talk, we will talk about the so-called defect operator associated to each invariant subspace, which has attracted much attention currently. It in principle contains all the information about the invariant subspace, but the problem is how to decipher. We will discuss some recent results, together with open problems.

Loukas Grafakos, University of Missouri, *Are L^2 -bounded homogeneous singular integrals necessarily L^p -bounded?*

We examine whether L^2 -bounded Calderon-Zygmund singular integrals with rough kernels must necessarily be L^p -bounded for some p not equal to 2.

Anatolii Grinshpan, Oklahoma State University, *Cosine expansions with nonnegative coefficients.*

I will discuss a family of polynomials having a certain positivity property. The study is motivated by the de Branges treatment of Loewner's differential equation.

Slim Ibrahim, McMaster University, *Global Solutions for a class of 2D NLS equations*

In this work, we define a new criticality notion for solutions of a 2D NLS equation with exponential type nonlinearity. We prove global well-posedness in the subcritical and critical cases, while ill-posedness is shown in the supercritical case.

Ning Ju, Oklahoma State University, *Dissipative 2D Quasi-geostrophic Equations.*

The two dimensional Quasi-Geostrophic Equations, either invicid or dissipative, are special cases of the general quasi-geostrophic approximations for the atmosphere and ocean flow with small Rossby and Ekman numbers. They were also proposed by Peter Constantin and Andrew Majda, etc as low dimensional model equations for mathematical study of possible development of singularity in smooth solutions of unforced incompressible three dimensional fluid equations. Indeed, the invicid 2D QGE is remarkably analogous to the 3D Euler Equations. The dissipative QGE at the critical dissipative rate is the right analogue of the 3D Navier-Stokes equations. The wellposedness theory of this system is thus a very challenging and interesting one. Recently, there has been some progress made, and still many problems remain open. In this talk, I will discuss some of the most important results obtained recently.

Priscilla S. Macansantos, University of the Philippines Baguio, *Volterra Integral Inclusions: Existence of Solutions*

We consider a Volterra integral inclusion of the form $x(t) \in f(t) + \int_0^t g(t, s, x(s))F(s, x(s))ds$, $0 \leq t \leq T$, where $F(s, x(s))$ is set-valued. By imposing a boundedness condition, convexity and a semicontinuity condition on the map F , we show that Kakutani's Fixed Point Theorem is applicable on the appropriate Banach space. In this way, existence is proven by way of a Fixed Point Theorem argument.

Diego Maldonado, University of Maryland, *Quasi-conformal maps with convex potentials.*

We give characterizations of quasi-conformal mappings (qcm) with convex potentials and use them to prove that for any set $E \subset \mathbb{R}^n$ of Hausdorff dimension less than 1 there is a qcm with

convex potential whose Jacobian vanishes/blows-up at every point of E . This is joint work with Leonid Kovalev.

Virginia Naibo, Rose-Hulman Institute of Technology, *Failure of an endpoint inhomogeneous Strichartz estimate.*

We prove that for the two-dimensional inhomogeneous Schrödinger equation $i\partial_t u + \Delta u = F$ with $u(x, 0) = 0$, the estimate $\|u\|_{L^2 L^\infty} \leq C \|F\|_{L^2 L^1}$ fails to be true. This complements the well-known result by Montgomery-Smith in the homogeneous case. This is joint work with Atanas Stefanov.

Generva Neumann, Kansas State University, *Cluster points and asymptotic values of planar harmonic functions.*

A planar harmonic function is a complex-valued harmonic function. The best understood planar harmonic functions are holomorphic functions. However, the behavior of planar harmonic functions can be vastly different from that of holomorphic functions. For example, analytic polynomials take every value a finite number of times. In contrast, the range of a harmonic polynomial can exclude an open region of the complex plane. The valence of a point w is the number of distinct zeros, not counting multiplicity, of $f(z) - w$. The image of a planar harmonic function is partitioned into regions of constant valence by the cluster set joined to the image of the critical set. This result will be reviewed briefly and examples given. Note that while this result holds vacuously if the image is a subset of the partitioning set (for example, $f(z) = e^z$), there are other types of planar harmonic functions (not just polynomials) where the function has positive valence off of the partitioning set. A sufficient condition for a cluster point of a planar harmonic function to be an asymptotic value will be given, based on this partitioning result. A sufficient condition for the cluster set of a planar harmonic function to have non-empty interior will also be discussed. An example will be given of a planar harmonic function where the image of the critical set is not closed and such that the cluster set has non-empty interior and is a proper subset of the image. This is in contrast to previous examples where the cluster set either had empty interior or filled the entire complex plane and where the image of the critical set was closed.

Mohammad Riazi-Kermani, Fort Hays State University

A potential counter-example for the $3n + 1$ conjecture is a natural number whose orbit does not include the integer 1. We show that it is possible to break a large integer into parts and iterate each part separately. This iteration by parts method is used to predict the behavior of large integers from the orbit of smaller parts. We investigate the binary structure of a potential counter-example by using iteration by parts. This method eliminates the need for iterating all those integers which are not candidates for a potential counter-example.

Cristian Rios, Trinity College, *The Dirichlet Problem for Infinitely Degenerate Quasilinear Equations.*

We prove interior a priori estimates for solutions of a class of infinitely degenerate elliptic quasilinear equations in divergence form. The conditions on the coefficients are nearly optimal. We use these estimates to obtain existence, uniqueness and regularity of weak solutions to the continuous Dirichlet problem. In two dimensions these results yield an interesting application to the regularity of degenerate elliptic Monge Ampere equations. This work has been done in collaboration with E.Sawyer from McMaster University and R. Wheeden from Rutgers University.

Caroline Sweezy, New Mexico State University, *Weights and integrability for parabolic and elliptic equations*.

Results on the improvement of integrability for the gradient of a solution to $(\frac{\partial}{\partial t} - L)u = \nabla \cdot f$ on a domain Ω in \mathbb{R}^{n+1} can be used to obtain sufficient conditions for two measures, μ and ν , so that $\|\nabla u\|_{L^q(\Omega, d\mu)} \leq C\|f\|_{L^p(\Omega, d\nu)}$ for a restricted range of p and q . The situation for elliptic operator solutions is also discussed.

Erin Terwilleger, University of Connecticut, *Hankel Operators and Product VMO*.

We consider the so called "little" Hankel operators on the Hardy space H^2 of functions on the bi-torus. These operators act on H^2 by pointwise multiplication by a function called the symbol, and then projecting onto the subspace of square integrable functions which is anti-holomorphic in each coordinate.

The main result is that the Hankel operators are compact if and only if the anti-holomorphic projection of the symbol is in the space dubbed product VMO. This is closely tied to a result of Ferguson and Lacey which states that the Hankel operator is bounded if and only if the projection of the symbol is in product BMO. There is also a formulation in terms of commutators.

Ignacio Uriarte-Tuero, *Improved Painleve removability for planar quasiregular mappings*.

The classical Painleve problem (characterize geometrically the sets of zero analytic capacity) has been recently solved by Tolsa (with previous partial results by Guy David, etc.). It is natural to try to understand the analogous problem in the quasiconformal world, i.e. understand the removable sets for bounded solutions of the Beltrami equation. I will present the results of a joint work with Astala, Clop, Mateu and Orobitg. It is known that not all compact sets of sigma-finite length in the plane are removable for bounded analytic functions (1 is the critical dimension for this problem). One of our main results is that, somewhat surprisingly, for the analogous quasiconformal problem (removability for bounded K-quasiregular mappings), all sets of sigma-finite measure at the critical dimension are removable.

The techniques come from complex analysis and quasiconformal mappings (conformal welding, integral means estimates, Makarov's compression and expansion for conformal mappings), multifractal analysis, nonlinear potential theory (Riesz and Bessel capacities), harmonic analysis (Calderón-Zygmund theory, Hörmander-Mihlin multiplier theorem), geometric measure theory, etc. I will try to make the talk as self-contained as possible.

Lianwen Wang, Central Missouri State University, *Applications of Nonsmooth Analysis to Optimal Control*

This talk focuses on optimal control problems of dynamical systems described by constrained functional differential inclusions. First, a sequence of discrete optimization problems to the original continuous optimization problems is constructed, then some necessary optimality conditions of discrete optimization problems are derived using nonsmooth analysis. Finally, by passing to the limit in the discrete necessary optimality conditions, the necessary optimality conditions of continuous optimal control problems are obtained, these results are expressed in terms of generalized differentiation for nonsmooth sets, functions, and set-valued mappings.

Lei Zhang, University of Florida, *Blowup solutions of some nonlinear elliptic equations involving exponential nonlinearities*.

In conformal geometry and several fields of physics, the blowup analysis of the equation $\Delta u + V(x)e^u = 0$ in $B_1 \subset \mathbb{R}^2$ has led to interesting results. In 1999 Li gave a uniform asymptotic estimate of a sequence of blowup solutions near an isolated blowup point. In this talk I present an improvement of Li's result to the sharp form by the moving sphere method.