

p -HARMONIC EQUATIONS

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The central theme running through the 2003 Prairie Lectures is the class of nonlinear PDEs whose prototype is the p -harmonic equation

$$\operatorname{div} |\nabla u|^{p-2} \nabla u = \operatorname{div} f, \quad 1 < p < \infty$$

A wider and more unifying framework will be established by means of differential forms and elliptic complexes, within which an associated q -harmonic dual system will exist, $p+q = pq$. The selection of topics reflects years of our interaction with scholars and scientists in several disciplines of mathematics: geometric function theory, calculus of variations, nonlinear elasticity, composites with micro-structures, and so forth. Thus, p -harmonic analysis will tell us something about the existence of elastic deformations, their regularity and topological behavior. While the Sobolev space $W^{1,p}(\Omega)$ is considered the natural domain of the p -harmonic operator, we shall depart from this space and move into the realm of so-called *very weak solutions* where important new applications lie. Viscosity solutions of the ∞ -Laplacian will emerge in the lecture of Juan Manfredi. Participants may wish to consult with John Lewis about the parabolic case as well. Our study of the p -harmonic equations beyond their natural domain of definition brings us closer to new methods of PDEs such as *nonlinear commutators* of singular integrals, *Hodge decomposition* and much more. One might possibly say that the p -harmonic analysis is an excellent source of ideas upon which we build a viable theory of "nonlinear singular integrals", like Riesz transforms for the Laplacian. We shall face the truly fascinating task of capturing the fundamental role of the p -harmonic operators in nonlinear problems, a virtue Riesz transforms do not possess. However, we shall not leave behind the linear theory of singular integrals. Every effort will be made to reduce to a minimum the technical aspects in the interest of mathematical insights.

Welcome to the III Prairie Analysis Seminar