Holomorphic Dynamics on bounded domains.

Abstract

Let Δ^n be a unit polydisc in $\mathbb{C}^n$ and let f be a holomorphic self map of Δ^n. In case $n = 1$, it is well known, by Schwarz's lemma, that f (not the identity) has at most one fixed point in the unit disc, Δ. Moreover, if f has one fixed point in Δ, then any (hyperbolic) disc centered at such a point is mapped into itself by f. If f has no fixed points in $Delta$, there exists a unique boundary point, say $x \in \partial\Delta$, such that every horocycle $E(x, R)$ of center x and radius $R > 0$ (the boundary relative of hyperbolic disc) is sent into itself by f. Such a point is called the *Wolff point of* f. In this talk we propose a definition of Wolff points for holomorphic maps defined on a (not necessarily strongly convex) bounded domain of $\mathbb{C}^n$.In particular we characterize the set of Wolff points, $W(f)$, of a holomorphic self-map f of the polydisc in terms of the properties of the components of the map f itself. Moreover we study the relationship between $W(f)$ and the target set of f, $\Gamma(f)$, namely the set of the limit points of the sequence of the iterates of f.