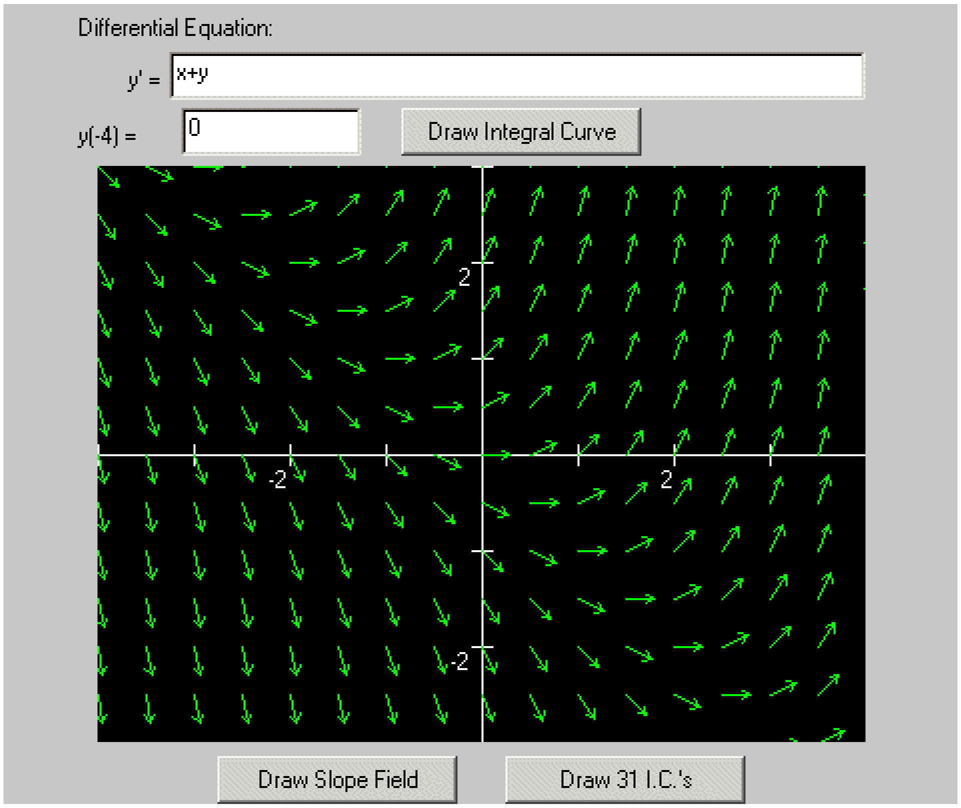
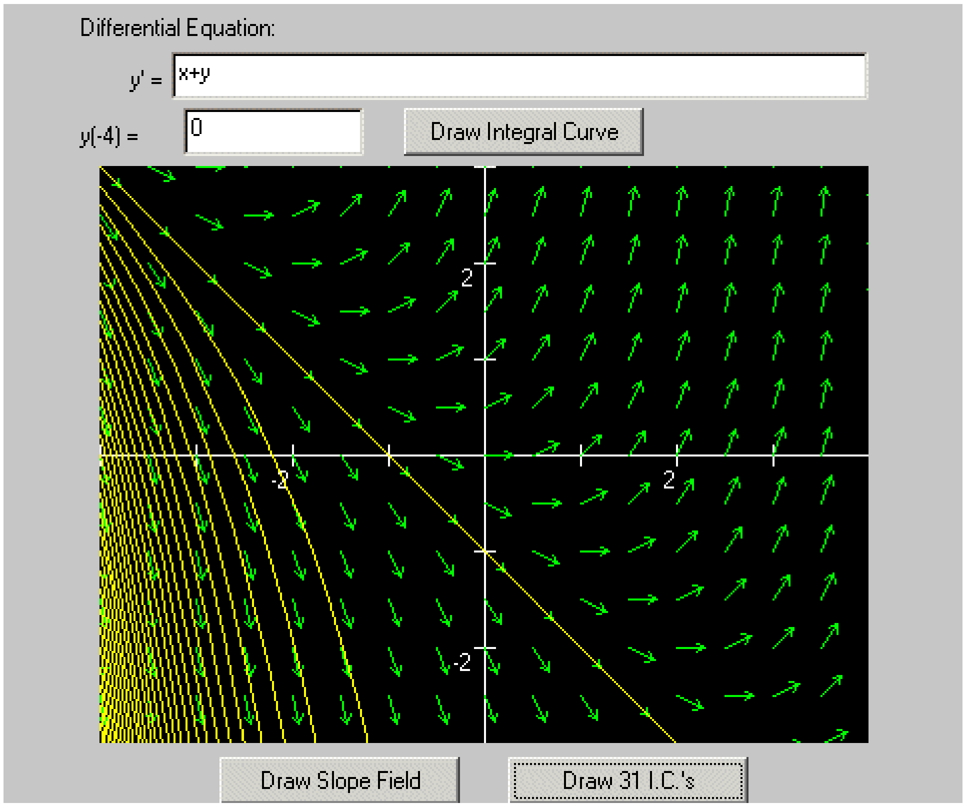
# Slope Fields Lab

Consider a differential equation $\frac{dy}{dx} = f(x,y)$. Since the derivative is the slope of the tangent line, we interpret this equation geometrically to mean that at any point in the plane $(x,y)$, the tangent line must have slope $f(x,y)$. We illustrate this with a slope field, a graph where we draw an arrow indicating the slope at a grid of points $(x,y)$. The slope field for $\frac{dy}{dx} = x + y$ is illustrated below.




The solution to a differential equation is a curve that is tangent to the arrows of the slope field. Since differential equations are solved by integrating, we call such a curve an integral curve. The picture at the top of the next page illustrates some of the integral curves for $\frac{dy}{dx} = x + y$. You can see there are a lot of possible integral curves, infinitely many in fact. This corresponds to the fact that there are infinitely many solutions to a typical differential equation. To specify a particular integral curve, you must specify a point on the curve. Once you specify one specific point, the rest of the curve is determined by following the arrows.




In this lab, you will experiment with the slope fields and integral curves for a variety of different equations. The goal is to get a geometric concept of what a differential equation means, to go along with the algebraic techniques to solve such equations. Launch the slope field lab from the class web site (www.math.ksu.edu/math240). This brings up an applet where you can type in differential equations and see their slope fields and integral curves. When you type in the differential equation, you must use * for multiplication, ^ for powers, and functions (such as sin(x), exp(x), log(x), etc.) must use parentheses. Once you type in a formula, you should press the "Draw Slope Field" button to draw the slope field (nothing will happen on screen until you press a button). If you've typed in a formula that the machine can't parse, it won't produce an error message but will just sit there until you correct the formula. The "Draw 31 I.C.'s" button will draw the integral curves passing through the points (-4, -3), (-4, -2.8), (-4, -2.6), … , (-4, 3) as illustrated above. You can also draw a single integral curve passing through the point (-4, $y_0$) by entering the value $y_0$ in the "y(-4) =" text field and pressing the "Draw Integral Curve" button next to it. Note that the "Draw Slope Field" button clears old results from the screen, while the other buttons just add new curves to the picture (so if the screen gets too confusing, press "Draw Slope Field" to clear it up).

The first pair of problems just ask you to play around with a couple of slope fields and initial conditions to get started and to make sure you understand the connection between the initial condition y(-4) = $y_0$ and the geometric picture of the integral curve.

1. Let $y' = \sin(x) + \cos(y)$. Find a value for y(-4) so that -3 < y(4) < 0.

2. Let $y' = 2*y - x$. Find a value for y(-4) so that -3 < y(4) < 3.

The next problem illustrates that the solution to a differential equation may not be defined over the whole real line, but may only exist in some particular interval.

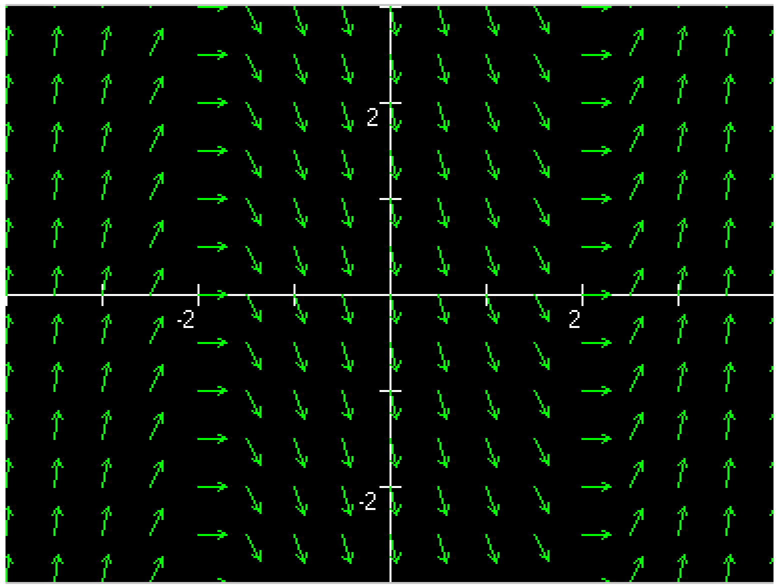
3. Let $y' = -(x+2)/y$. **Do not try to draw the integral curves.** If you try to draw the integral curves, the applet will take a very long time to respond (or the machine will just hang) and you will probably need to reboot. Look at the slope field and write a sentence or two to explain why the machine ultimately runs into trouble in trying to follow the arrows (while it is technically correct to say the machine runs into trouble because the programmer didn't design the applet carefully enough, that isn't the answer we're looking for).

In the final 4 problems, you will develop some feeling for the connections between the slope field and certain types of differential equations. First you will consider some examples, and then you can use the ideas you've developed to try to recognize a couple of equations by their slope fields.

4. Look at several examples of slope fields of the form $y' = f(x)$ where the right-hand-side doesn't depend on y. Press the "Draw 31 I.C.'s" button to look at the integral curves for these slope fields. You should observe a pattern in such fields. Write one or two sentences to describe how you can tell if a slope field corresponds to such an equation.

5. Look at several examples of slope fields of the form $y' = f(y)$ where the right-hand-side doesn't depend on x (such equations are called "autonomous"). You should observe a pattern in such fields. Press the "Draw 31 I.C.'s" button to look at the integral curves for these slope fields. Do the integral curves have the same sort of pattern as for the fields considered in problem 4? Write one or two sentences to describe how you can tell if a slope field corresponds to such an equation.

6. At the top of the next page is the slope field for y' = f(x,y). Where is the function f(x,y) positive, negative, and zero? Is f(x,y) a function of x alone, y alone, or a function of both variables together? Find a function f(x,y) whose slope field looks like this?




7. Below is the slope field for y' = f(x,y). Where is the function f(x,y) positive, negative, and zero? Is f(x,y) a function of x alone, y alone, or a function of both variables together? Find a function f(x,y) whose slope field looks like this?