Review for Final Exam - Spring 2002 - selected problems - **Spring 1999**

1. Find the general solution: $y' = y^3 + y$.

2. Find the general solution to $x'' - 3x' + 2x = e^t$.

3. Without solving the equation, graph several solution curves for the differential equation:

$$\frac{dP}{dt} = P^2(P+2)(P-4)$$

Find the equilibrium points and classify each as stable or unstable. Make sure you draw at least two solution curves between each pair of equilibrium points.

4. Use the Euler method with stepsize $h = .5$ to estimate $y(1)$ for the initial value problem:

$$\frac{dx}{dt} = tx + y; \quad x(0) = 1$$

$$\frac{dy}{dt} = xy + t; \quad y(0) = 0$$

5. Solve $x^2y'' - xy' + y = 0$. Here you may assume $x > 0$.

6. Find the general solution: $xy' + 2y = x$.

7. Solve: $x'' + 9x' + 20x = f(t), \quad x(0) = 4, x'(0) = 0$. Part of your solution should be left as a convolution integral involving the function $f(t)$, but you should find explicitly the other terms of the solution.

8. A 1 kg mass is attached to a spring which causes the spring to stretch 4.9 m. This is attached to a damping mechanism with damping constant 2 kg/sec. The mass is given no initial displacement but is set into motion with an initial velocity of 1 m/sec. At time $t = 2$ the

mass is subjected to an impulse force. (That is, the forcing function is $\delta(t-2)$). Find $x(t)$, the function which gives the position of the mass at time t.

9. Find the general solution: $2xy + cosx = -x^2\frac{dy}{dx}$.

10. Find a solution of the form $y(x) = \Sigma_{n=0}^{\infty} a_n x^n$ for the initial value problem $y'' + xy = 0,\ y(0) = 1,\ y'(0) = 0$. Include at least the first three non-zero terms.

11. (a) Determine the real singular points of the differential equation:

$$(x^2+1)(x-1)^2(x-2)^2y'' + (x-1)y' + y = 0$$

(b) For each of the singular points you found in (a) classify it as a regular singular point or an irregular singular point.

(c) Suppose it is desired to solve this equation with a power series of the form $y(x) = \Sigma_{n=0}^{\infty} a_n(x+1)^n$. Give a lower bound for the radius of convergence of such a series solution.

12. Find the series solution (centered at 0) corresponding to the larger root of the indicial equation for the differential equation

$$x^2y'' + 3xy' + (x+3)y = 0$$

Include at least the first three non-zero terms.

13. Solve: $y' = \frac{2y^4+x^4}{xy^3}$.

14. Solve using Laplace transform: $x'' + 16x = e^{2t}$, $x(0) = 0, x'(0) = 0$.