Written Assignment #15: Euler Equations

Due 5:00pm Tuesday, December 9, 2003

You are encouraged to collaborate with your colleagues. For credit, however, your final write-up must be done individually. Show all your work and make your presentation comprehensible.

1. Find the general solution to the following inhomogeneous Euler equation. Assume that $x > 0$.
$$x^2y'' - 3xy' + 4y = x^3.$$
(Hint: use the variation of parameters formula.)

2. A basic partial differential equation is Laplace's equation in two-dimensions:
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$
where $u = u(x, y)$ is a function of both x and y.

   (a) Rewrite Laplace's equation in polar coordinates. Recall for polar coordinates that the modulus is
   $$r = \left(x^2 + y^2\right)^{\frac{1}{2}}$$
   and the argument is
   $$\theta = \begin{cases} \arctan(y/x), & x \geq 0; \\ \arctan(y/x) + \pi, & x < 0 \text{ and } y \geq 0; \\ \arctan(y/x) - \pi, & x < 0 \text{ and } y < 0. \end{cases}$$
   You will also need to use the chain rule for functions of several (two) variables from your Calculus III course.

   (b) Find the general rotationally symmetric solution to Laplace's equation in two-dimensions. A rotationally symmetric solution is independent of θ, so its values do not change with θ; it can be thought of as a function of r alone.