

Extra Credit Assignment #4: Series Solutions

Due in the review session Tuesday, December 9, 2003

You are encouraged to collaborate with your colleagues. For credit, however, your final write-up must be done individually. Show all your work and make your presentation comprehensible.

For this assignment, you just need to solve the following problems.

1 Problems

1. Find the recurrence relation and general series solution, centered at $x_0 = 0$, for the following differential equations. Give a lower bound for the radius of convergence for each solution.

(a) $(x^2 - 1)y'' + 4xy' + 2y = 0$.

(b) $(x^2 + 2)y'' + 4xy' + 2y = 0$.

2. Find a series solution, centered at $x_0 = 1$ for the following initial value problems. Determine the radius of convergence for each solution.

(a) $y'' + (x - 1)y' + y = 0$; $y(1) = 2, y'(1) = 0$.

(b) $(x^2 - 2x + 2)y'' - 4(x - 1)y' + 6y = 0$; $y(1) = 2, y'(1) = 0$.

Hint: Make the change of variables $t = x - 1$ and find the series solution of the form $\sum_{n=0}^{\infty} a_n t^n$. Then replace t with $x - 1$.