

Name: _____

Recitation Instructor and Time: _____

Studio College Algebra – Exam 3
July 17, 2009

1. Rewrite the formula $P = \frac{x^2}{7}$ by taking the logarithm of both sides.

$$\ln P = \ln\left(\frac{x^2}{7}\right)$$

$$\ln P = \ln x^2 - \ln 7$$

$$\boxed{\ln P = 2 \ln x - \ln 7}$$

$$\log P = \log\left(\frac{x^2}{7}\right)$$

$$\text{or } \log P = \log x^2 - \log 7$$

$$\boxed{\log P = 2 \log x - \log 7}$$

2. Rewrite the formula $y = 1.043e^x$ by taking the logarithm of both sides.

$$\ln y = \ln(1.043e^x)$$

$$\ln y = \ln 1.043 + \ln e^x$$

$$\ln y = \ln 1.043 + x \ln e$$

$$\boxed{\ln y = \ln 1.043 + x}$$

$$\log y = \log 1.043 + \log e^x$$

$$\text{or } \boxed{\log y = \log 1.043 + x \log e}$$

3. If $\log(x) = 4.3$ and $\log(y) = 9.6$, what is $\log(x\sqrt{y})$?

$$\begin{aligned}\log(x\sqrt{y}) &= \log x + \log \sqrt{y} \\ &= \log x + \log y^{1/2} \\ &= \log x + \frac{1}{2}\log y \\ &= 4.3 + \frac{1}{2}(9.6) = 4.3 + 4.8 = \boxed{9.1}\end{aligned}$$

4. Solve $3 + 5^x = 4$

$$\begin{aligned}\cancel{3} + 5^x &= 1 \\ \ln 5^x &= \ln 1 \\ x \ln 5 &= 0 \\ x &= \frac{0}{\ln 5} = 0 \\ \boxed{x=0}\end{aligned}$$

5. Solve $\ln(x+2) + 9 = 10$.

$$\begin{aligned}\ln(x+2) &= 1 \\ e^1 &= x+2 \\ \boxed{e-2=x} &\text{ or } \boxed{x \approx .71828\dots}\end{aligned}$$

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6. What is the future value, in 10 years, of an initial investment of \$22,000 at an annual interest rate of 5%, compounded weekly?

$$FV = PV \left(1 + \frac{r}{n}\right)^{nt}$$

$$FV = 22,000 \left(1 + \frac{.05}{52}\right)^{520}$$

$$FV = \$36,263.16$$

7. Suppose the population of a certain town could be modeled by $P(t) = 3600e^{.00713t}$, where t represents the number of years since 1990.

a) What is the initial population of the town?

$$P(0) = \underline{3600 \text{ people}}$$

b) *During* what year will the population pass 4,100?

$$4100 = 3600 e^{.00713t}$$

$$\frac{4100}{3600} = e^{.00713t}$$

During 2008

$$\frac{41}{36} = e^{.00713t}$$

$$\ln\left(\frac{41}{36}\right) = .00713t$$

$$\frac{\ln\left(\frac{41}{36}\right)}{.00713} = t$$

$$t \approx 18.24$$

8. The revenue function for the sale of a certain product is given by $R(x) = -x^3 - 79x^2 + 1992x$, where x is the number of units sold. If the sale of 9 units leads to \$10,800 in revenue, what is another number of units that leads to the same amount of revenue?

$$\begin{array}{r}
 9) \quad -1 \quad -79 \quad 1992 \quad -10,800 \\
 \quad \downarrow \quad -9 \quad -792 \quad 10800 \\
 \hline
 \quad -1 \quad -88 \quad 1200 \quad 0
 \end{array}$$

9. Find a 4th degree polynomial with single roots at $x = -3$ and $x = 5$, and a double root at $x = 1$. Write the polynomial in standard form $a_n x^n + \dots + a_1 x + a_0$.

$$\begin{aligned}
 & (x+3)(x-5)(x-1)(x-1) \\
 & = (x^2 - 2x - 15)(x^2 - 2x + 1)
 \end{aligned}$$

$$\begin{aligned}
 & = x^4 - 2x^3 - 15x^2 \\
 & \quad - 2x^3 + 4x^2 + 30x \\
 & \quad \quad + x^2 - 2x - 15
 \end{aligned}$$

$$= \boxed{x^4 - 4x^3 - 10x^2 + 28x - 15}$$

10. Given that $x = 5$ is a root of the polynomial $P(x) = x^3 - 125$, find all other roots, both real and complex, of $P(x)$.

$$\begin{array}{r|rrrr} 5 & 1 & 0 & 0 & -125 \\ & \downarrow & 5 & 25 & 125 \\ \hline & 1 & 5 & 25 & 0 \end{array}$$

$$x^2 + 5x + 25$$

$$\frac{-5 \pm \sqrt{25 - 4(1)(25)}}{2}$$

$$= \frac{-5 \pm \sqrt{75}i}{2}$$

$$= \left[\frac{-5}{2} \pm \frac{5\sqrt{3}i}{2} \right]$$

11. The number of gizmos (in thousands) demanded each year is given by the formula $D(x) = 5 + 3 \log(x + 7)$, where x represents the number of years after 1970, and $x > 0$. How many gizmos were demanded in the year 1994?

$$D(24) = 5 + 3 \log(24 + 7)$$

$$= 5 + 3 \log(31)$$

$$= 9.474$$

$$\boxed{9.474 \text{ gizmos}}$$

12. The revenue function for a certain product is given by $R(x) = 128,000x - 0.2x^3$, where x is the number of units sold. What values of x result in zero revenue?

$$0 = 128,000x - 0.2x^3$$

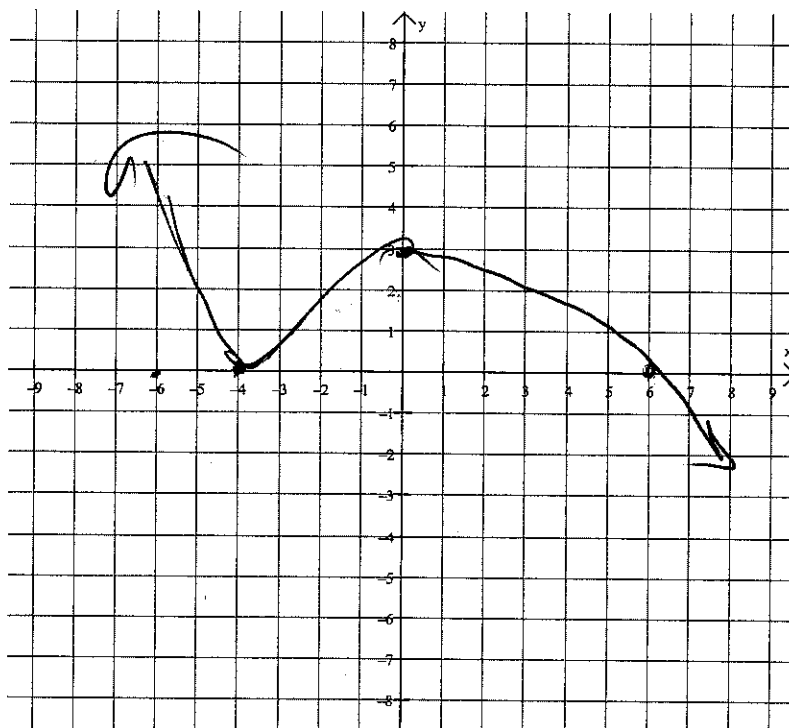
$$0 = .2x(640,000 - x^2)$$

$$0 = .2x(800 - x)(800 + x)$$

$$x = 0 \text{ or } x = 800 \text{ units}$$

13. Sketch a polynomial on the grid below with the given characteristics:

- Odd degree
- Negative leading coefficient
- Positive constant term
- A double root at $x = -4$
- A single root at $x = 6$



14. What is the horizontal asymptote of the function $h(x) = e^x + 3$? Explain how you arrived at your answer.

$$\boxed{y=3}$$

(The asymptote of $y = e^x$ is shifted vertically +3 units)

15. What is the domain of the function $k(x) = 2\ln(4-x) + 3$? Show all work.

$$4 - x > 0$$

$$4 > x$$

$$\boxed{x < 4}$$

16. The volume of a rectangular prism can be described by the formula $V(x) = 784x - 112x^2 + 4x^3$, where x is the height of the prism in inches.

- a) Find all the roots of $V(x)$. Are any of the roots repeated roots?

$$4x(14 - x)(14 - x) = 0$$

$$x = 0, \quad x = 14$$

↑ repeated

- b) What is $V(10)$?

$$\begin{aligned} V(10) &= 784(10) - 112(10)^2 + 4(10)^3 \\ &= \boxed{640 \text{ sq. inches}} \end{aligned}$$